INTEGERATION

If $F'(x) = f(x)$ for all x - internal then $F(x)$ is called integral or Anti derivative for all values of $f(x)$ for that interval.

i.e $\frac{d}{dx} F(x) = f(x)$

$d\,F(x) = f(x)\,dx.$

$\int d\,F(x) = \int f(x)\,dx.$

$\therefore \int f(x)\,dx = F(x) + c.$

$F(x) \longrightarrow$ Integral

$\int f(x)\,dx = F(x) + c \longrightarrow$ constant of integral

$dx \longrightarrow$ operator.

$f(x) \longrightarrow$ Integrated function

$\int \longrightarrow$ sign of integration

$\int dx = x.$

dx — dirver, $f(x)$ passes

$dx \longrightarrow x$

$\int f(x) \longrightarrow x$

$f(x) \longrightarrow x$

$f(x)\,dx \longrightarrow x.$

$\int dx = x$

$\int x^0 = x$

$\frac{x^{0+1}}{1} = x$

$\frac{x^1}{1} = x$

$\boxed{x = x}$

TYPES OF INTEGRATION

Indefinite (no limit)

$\int f(x)\,dx = F(x) + c$

Definite. (Having limit)

$\int_a^b f(x)\,dx$

a → lower limit

b → upper limit

Definite Integral.

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b$$

upper - lower

$F(b) - F(a)$

Ex

$$\int_2^3 f(4x)\,dx = \left[F(4x)\right]_2^3$$

$4(3) - 4(2) =$

$\Rightarrow 4$

(i) $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c$

where c → constant of indefinite.

(ii) $\int (ax+b)^n\,dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

) $\int [f(x)]^n f'(x)\,dx = \dfrac{[f(x)]^{n+1}}{n+1}$

(iv) $\int \frac{1}{f(x)} f'(x)\,dx = \ln f(x) + c.$

Evalute the following indefinite Integral

Q1. $\int (x^2 + 4x + 13)\,dx$

Sol $\int (x^2 + 4x + 13)\,dx.$

$\Rightarrow \int x^2\,dx + \int 4x\,dx + \int 13\,dx$

$\Rightarrow \frac{x^{2+1}}{2+1} + 4\int x\,dx + 13\int dx.$

$\Rightarrow \frac{x^3}{3} + (4)\frac{x^2}{2} + 13x + c.$

$\Rightarrow \frac{x^3}{3} + 2x^2 + 13x + c$ Ans.

Q2 $\int (3x^4 - 5x^3 - 4x^2 - 2)\,dx$

Sol $\int (3x^4 - 5x^3 - 4x^2 - 2)\,dx.$

$\Rightarrow 3\int x^4\,dx - 5\int x^3\,dx - 4\int x^2\,dx - 2\int dx$

$\Rightarrow 3\frac{x^5}{5} - 5\frac{x^4}{4} - 4\frac{x^3}{3} - 2x + c$

Ans

Sol

$\Rightarrow \int x^2(x^2-4)\,dx$

$\Rightarrow \int (x^4-4x^2)\,dx.$

$\Rightarrow \int x^4\,dx - 4\int x^2\,dx.$

$\Rightarrow \left\{ \frac{x^{4+1}}{5} - 4\left(\frac{x^3}{3}\right) + c \right.$

$\Rightarrow \frac{x^5}{5} - \frac{4}{3}x^3 + c.$

Q⑤ $\int \sqrt{2x+5}\,dx \longrightarrow (i)$

Let

$2x+5 = u \longrightarrow (ii)$

Diff w.r.t (ii) w.r.t (x).

$2\frac{d}{dx}(x) + \frac{d5}{dx} = \frac{du}{dx}.$

or $\frac{du}{dx} = 2$

$dx = \frac{1}{2}du.$

$\therefore$ Equ (i) $\Rightarrow \int \sqrt{u}\left(\frac{1}{2}du\right).$

$\Rightarrow \frac{1}{2}\int (u)^{1/2}\,du.$

$\Rightarrow \frac{1}{2} \times \frac{(u)^{3/2}}{3/2} + c.$

$\Rightarrow \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}(u)^{3/2} + c.$

$\Rightarrow \frac{1}{3}(2x+5)^{3/2} + c.$ Ans

$\int \sqrt{2x+5}\,dx$

$\Rightarrow \int \sqrt{2x+5}\,\frac{d(2x+5)}{dx}\,dx$

$\frac{1}{2}\int \sqrt{2x+5}\,(2)\,dx$

$\frac{1}{2} \cdot \frac{(2x+5)^{3/2}}{3/2} + c$

$3(2x+5)^{3/2} + c$

8.3

Q4 (i) $\int e^{2-3x}\,dx = ?$

Sol

$\Rightarrow \int e^{2-3x}\,dx$

Put $2-3x=u$

$-3dx = du$

$dx = -\frac{du}{3}$

$\Rightarrow -\frac{1}{3}\int e^{u}\,du$

$\Rightarrow -\frac{e^{u}}{3}+c$

$\Rightarrow -\frac{e^{2-3x}}{3}+c$

Q2

Q(v) $\int \frac{x^2\,dx}{e^{2x^3+3}}$

Let $2x^3+3=u$

$6x^2dx = du$

$x^2\,dx = \frac{du}{6}$

$\Rightarrow \frac{1}{6}\int \frac{1}{e^{u}}\,du$

$\Rightarrow \frac{1}{6}\int e^{-u}\,du$

$\Rightarrow \frac{1}{6}\,\frac{e^{-u}}{-1}+c$

$\Rightarrow -\frac{1}{6}\,e^{-u}+c$

$\Rightarrow -\frac{1}{6}\,e^{-(2x^3+3)}+c$

$\frac{d}{dx}a^x = a^x \ln a$

Q $y = a^x$

$= e^{\ln a^x}$

$= e^{x\ln a}$

$\frac{dy}{dx} = e^{x\ln a} \cdot \frac{d}{dx} x\ln a$

~~$= e^{x\ln a}\left[x\frac{d}{dx}\ln a + \ln a\frac{d}{dx}x\right]$~~

~~$\Rightarrow e^{x\ln a}\, x$~~

$= e^{x\ln a} \frac{d}{dx}(x\ln a)$

$= e^{x\ln a}\left[\ln a \frac{d}{dx} x\right]$

$= e^{x\ln a} \ln a$

$\frac{dy}{dx} = a^x \ln a$

Q $\frac{dy}{dx} y = ?$ $\quad y = 5^x$

$y = 9^x$

$y = b^x$

$y = 7^x$

Q $\int a^{2y}\, dy$

$\Rightarrow \int e^{\ln a^{2y}} dy$

$\Rightarrow \int e^{2y\ln a} dy$

~~put~~

$2y \ln a = u$

$$2\ln a\, dy = du$$

$$dy = \frac{du}{2\ln a}.$$

$$\Rightarrow \frac{1}{2\ln a}\int e^{u} du.$$

$$\Rightarrow \frac{1}{2\ln a} e^{u} + c$$

$$\Rightarrow \frac{1}{2\ln a} e^{2y\ln a} + c.$$

$$\Rightarrow \frac{1}{2\ln a} a^{2y} + c.$$

Area

Q $\int a^{2y}\, dy = ?$

Sol Let $2y = u$

$2\, dy = du$

$$\Rightarrow \frac{1}{2}\int a^{u}\, du$$

$\Rightarrow$ Using $\boxed{\int a^{x} dx = \frac{a^{x}}{\ln a} + c.}$

$$\Rightarrow \frac{1}{2}\frac{a^{u}}{\ln a} + c$$

$$\therefore \frac{1}{2}\frac{a^{2y}}{\ln a} + c$$

8.3

4 Q (iv) $\int \frac{e^{\frac{1}{u}}}{u^2} du = ?$

$\Rightarrow \int \frac{e^{\frac{1}{u}}}{u^2} du$

Let $\frac{1}{u} = t$

$(-1)\frac{u^{-2} du}{u^{-2}} = dt$

$-\frac{1}{u^2} du = dt$

$\therefore \int e^{t}(-dt)$

$\Rightarrow (-1)\int e^{t} dt$

$\Rightarrow (-1)e^{t} + c$

$\Rightarrow -e^{\frac{1}{u}} + c$ Ans

Q $\int e^{x}(2e^{3k}-5)^{2/5} dx$

$(2e^{3k}-5)^{2/5}\int e^{x} dx$

$\because k =$ constant

i.e. $(2e^{3k}-5)^{2/5} =$ constant.

$(2e^{3k}-5)^{2/5}\int e^{x} dx$

$\boxed{(2e^{3k}-5)^{2/5} e^{x} + C}$ Ans

Chpt# 8.
ANTIDERIVATIVE.

Formula:-

$\boxed{1}$ $$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \ \forall n = -1$$

where, c = constant of integration.

Q 1// $\int x^n dx$

$$= \frac{x^{3+1}}{3+1} + c$$

$$= \frac{x^4}{4} + c$$

Q:- $\int x^{-2} dx.$

$$= \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{x^{-1}}{-1} + c$$

$$= -\frac{1}{x} + c$$

Q:- $\int \sqrt{x}\, dx$

$$= \int (x)^{\frac{1}{2}} dx.$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{2x^{3/2}}{3} + c$$

Q:- $\int x^3 dx.$

$$\boxed{= \frac{x^4}{4} + c}$$

Let

$$y = \frac{x^4}{4} + c$$

diff w.r.t 'x'

$$\frac{dy}{dx} = \frac{\cancel{4} x^3}{\cancel{4}} + 0$$

$$\frac{dy}{dx} = x^3$$

$$\frac{dy}{\cancel{dx}} = x^3 dx$$

Q:- $\int x^{-4} dx.$

$$dy = -\frac{1}{3x^3} + c.$$

Note:-

$$\boxed{\int dx = x + c}$$

Proof:-

$$\int dx = \int 1.dx$$

$$= \int x^0 dx$$

$$= \frac{x^{1+0}}{1+0} + c$$

$$\boxed{\int dx = x + c}$$

Q:- $\int x^{4/5} dx$

$$= \frac{x^{4/5+1}}{4/5+1} + c$$

$$= \frac{x^{9/5}}{\frac{9}{5}}$$

$$= \frac{5}{9} x^{9/5} + c.$$

Q:- $\int (x)^{-2/3} dx.$

$$= \frac{x^{-2/3+1}}{-2/3+1} + c$$

$$= \frac{x^{\frac{1}{3}}}{\frac{1}{3}}$$

$$= 3x^{1/3} + c$$

Q:- $\int (x)^{-3/5} dx.$

$$= \frac{x^{-3/5+1}}{-3/5+1} + c$$

$$= \frac{5x^{2/5}}{2} + c.$$

Q:- $\int x^{1/7} dx = \frac{7x^{8/7}}{8} + c$

Properties

$\boxed{1}$ $\boxed{\text{If } \int k \cdot x^n dx = k \int x^n dx.}$

where, k = constant.

$\boxed{2}$ If

$\int [f(x) + g(x) + h(x) + u(x) + v(x)] dx = \int f(x) dx + \int g(x) dx$

$+ \int h(x) dx + \int u(x) dx + \int v(x) dx.$

Q:- $\int (x^3 + x^4 + x^2 + x^5 + 2x) dx$

$= \int x^3 dx + \int x^4 dx + \int x^2 dx + \int x^5 dx + 2 \int x dx.$

$\Rightarrow \frac{x^4}{4} + \frac{x^5}{5} + \frac{x^3}{3} + \frac{x^6}{6} + \cancel{2} \frac{x^2}{\cancel{2}} + c$

$\Rightarrow \boxed{\frac{1}{4} x^4 + \frac{1}{5} x^5 + \frac{1}{3} x^3 + \frac{1}{6} x^6 + x^2 + c}$ Ans

5, 6, 10, 7

Q:- $\int (x^{1/3} - 2x^{4/5} + x^{3/2} + \sqrt{x})\, dx.$

$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - 2\frac{x^{4/5+1}}{4/5+1} + \frac{x^{3/2+1}}{3/2+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \frac{x^{4/3}}{4/3} - 2\frac{x^{9/5}}{\frac{9}{5}} + \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + c$

$= \boxed{\frac{3}{4}x^{4/3} - \frac{10}{9}x^{9/5} + \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + c}$

8.1

Q 1 $\int (x^2 + 4x + 13)\, dx$

Sol

$= \int (x^2 + 4x + 13)\, dx$

$= \int x^2 dx + 4\int x\, dx + 13\int dx.$

$= \frac{1}{3}x^3 + \frac{4}{2}x^2 + 13x + c$

$= \frac{1}{3}x^2 + 2x^2 + 13x + c$ Ans

Q 11// $\int x(x^3+1)^2\, dx$

$\Rightarrow \int x(x^3+1)^2 dx.$

$\Rightarrow \int x(x^6+2x^3+1) dx$

$= \int (x^7+2x^4+x) dx$

$\Rightarrow \int x^7 dx + 2\int x^4 dx + \int x dx$

$= \frac{x^8}{8} + 2\frac{x^5}{5} + \frac{x^2}{2} + c$

$= \frac{1}{8}x^8 + \frac{2}{5}x^5 + \frac{1}{2}x^2 + c$

Q2 $\int (3x^4 - 5x^3 - 4x^2 - 2) dx.$

Sol

$\int (3x^4 - 5x^3) - 4x^2 - 2) dx$

$\Rightarrow 3\int x^4 dx - 5\int x^3 dx - 4\int x^2 dx - 2\int dx.$

$\Rightarrow \frac{3x^5}{5} + -\frac{5x^4}{4} - \frac{4}{3}x^3 - 2x + c$

$\Rightarrow \frac{3}{5}x^5 - \frac{5}{4} - \frac{4}{3}x^3 - 2x + c$

Q4 $\int x^2(x^2-4) dx$

Sol

$\Rightarrow \int x^2 (x^2-4) dx$

$\Rightarrow \int x^4 - 4x^2 dx$

$\Rightarrow \int x^4 - 4\int x^2 dx$

$\Rightarrow \frac{x^5}{5} - \frac{4}{3}x^3 + c$ Ans

Q8 $\int \frac{du}{u^2}$

Sol

$\Rightarrow \int \frac{du}{u^2}$

$\Rightarrow \int u^{-2} du$

On integrating

$\Rightarrow \frac{u^{-2+1}}{-2+1} + c$

$\Rightarrow -\frac{u^{-1}}{1} + c$

$\Rightarrow -\frac{1}{u} + c$ Ans

Q9 $\int \frac{6}{u^5} du = ?$

Sol

$\Rightarrow \int \frac{6}{u^5} du.$

$\Rightarrow 6\int u^{-5} du.$

$\Rightarrow 6\left[\frac{u^{-5+1}}{-5+1}\right] + c.$

$\Rightarrow 6\left(\frac{u^{-4}}{-4}\right) + c.$

$= -\frac{3}{2u^4} + c$ Ans

Q 10 $\int \frac{du}{\sqrt{a}}$

Q 11 $\int x(x^3+1)^2 dx.$

Sol

$\Rightarrow \int x\{x^3+1\}^2 dx$

$\Rightarrow \int x\{(x^3)^2 + 2(x^3)(1) + (1)^2\} dx$

$\Rightarrow \int x\{x^6 + 2x^3 + 1\} dx$

$\Rightarrow \int (x^7 + 2x^4 + x) dx$

$\Rightarrow \int x^7 dx + 2\int x^4 dx + \int x dx.$

$\Rightarrow \frac{x^8}{8} + 2\frac{x^5}{5} + \frac{x^2}{2} + c$

$= \frac{1}{8}x^8 + \frac{2}{5}x^5 + x^2\frac{1}{2} + c$

Ans

Exam pg 215

$$\int (ax^3 + bx^2 + cx + e)\,dx = ?$$

Sol

$$\int (ax^3 + bx^2 + cx + e)\,dx$$

$$\Rightarrow a\int x^3 dx + b\int x^2 dx + c\int x\,dx + e\int dx.$$

$$\Rightarrow \frac{a}{4}x^4 + \frac{b}{3}x^3 + \frac{c}{2}x^2 + ex + c.$$

Ans

FORMULAE.

$\boxed{12}$ $$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \quad \forall\, n \neq -1 \text{ where } c = \text{constant}$$

OWN

Q $$\int (3x+5)^{1/2} dx$$

$$\Rightarrow \frac{(3x+5)^{\frac{1}{3}+1}}{3\left(\frac{1}{3}+1\right)} + c$$

$$\Rightarrow \frac{3(3x+5)^{4/3}}{3\left(\frac{4}{3}\right)} + c$$

$$\Rightarrow \frac{(3x+5)^{4/3}}{4} + c$$

Q $$\int \sqrt{6x-5}\,dx.$$

$$\Rightarrow \int (6x-5)^{1/2} dx.$$

$$\Rightarrow \frac{(6x-5)^{\frac{1}{2}+1}}{6\left(\frac{1}{2}+1\right)} + c$$

$$\Rightarrow \frac{(6x-5)^{3/2}}{\left(\frac{3}{2}\right)\cancel{6}3} + c$$

$$\Rightarrow \boxed{\frac{(6x-5)^{3/2}}{9} + c}$$ Ans

6, 7, 10, 18

Q15// $\int \sqrt{2x+5}\, dx.$

Sol

$$\Rightarrow \int (2x+5)^{1/2}\, dx.$$

$$\Rightarrow \frac{(2x+5)^{\frac{1}{2}+1}}{2\left(1+\frac{1}{2}\right)} + c$$

$$\because \int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\Rightarrow \boxed{\frac{(2x+5)^{3/2}}{3} + c}$$ Ans

Q13 $\int (\sqrt{x}-1)^2\, dx$

$$\Rightarrow \int (x - 2\sqrt{x} + 1)\, dx$$

$$\Rightarrow \int x\, dx - 2\int x^{\frac{1}{2}}\, dx + 1\int dx.$$

$$\Rightarrow \frac{1}{2}x^2 - 2\,\frac{x^{3/2}}{3/2} + x + c$$

$$\Rightarrow \frac{1}{2}x^2 - \frac{4}{3}x^{3/2} + x + c$$ Ans

Q17// $\int \frac{(\sqrt{\theta}-1)^3}{\sqrt{\theta}} \cdot d\theta$ 15, 16.

$$\Rightarrow \int \left[\frac{(\sqrt{\theta})^3 - (1)^3 - 3(\sqrt{\theta})(1)(\sqrt{\theta}-1)}{\sqrt{\theta}}\right] dx$$

$$\Rightarrow \int \left[\frac{\theta^{3/2} - 1 - 3\theta^{1/2}(\theta^{1/2} - 1)}{\theta^{1/2}} \right] d\theta$$

$$\Rightarrow \int \left[\frac{\theta^{3/2} - 1 - 3\theta + 3\theta^{1/2}}{\theta^{1/2}} \right] d\theta$$

$$\Rightarrow \int \left[\frac{\theta^{3/2}}{\theta^{1/2}} - \frac{1}{\theta^{1/2}} - \frac{3\theta}{\theta^{1/2}} + 3\frac{\cancel{\theta^{1/2}}}{\cancel{\theta^{1/2}}} \right] d\theta$$

$$\Rightarrow \int \left(\theta - \theta^{-1/2} - 3\theta^{1/2} + 3 \right) d\theta$$

$$\Rightarrow \int \theta \, d\theta - \int \theta^{-1/2} d\theta - 3\int \theta^{1/2} d\theta + 3\int d\theta$$

$$\Rightarrow \frac{\theta^2}{2} - \frac{\theta^{1/2}}{(1/2)} - 3\frac{\theta^{3/2}}{(3/2)} + 3\theta + c.$$

$$\Rightarrow \frac{\theta^2}{2} - 2\theta^{1/2} - \frac{6}{3}\theta^{3/2} + 3\theta + c.$$

FORMULAE

$$\int f^n(x) f'(x)\, dx = \frac{f^{n+1}(x)}{n+1} + c. \quad \forall\, n \neq -1$$

$$\Downarrow$$

[For. Equivalent to Algebraic Substitution].

Q $\int (x^3+1)^5 (3x^2)\,dx.$

$\Rightarrow \dfrac{(x^3+1)^6}{6} + c.$

Q:- $\int (\sqrt{x^2-1})(2x)\,dx$

$\Rightarrow \int (x^2-1)^{1/2} (2x)\,dx$

$\Rightarrow \dfrac{2}{3} (x^2-1)^{3/2} + c$

Q:- $\int (x^4+6)^{3/2} \dfrac{4}{4} x^3\,dx$

$\Rightarrow \dfrac{1}{4} \int (x^4+6)^{3/2}\, 4x^3\,dx$

$\Rightarrow \dfrac{1}{4} \left[\dfrac{(x^4+6)^{5/2}}{5/2} \right] + c$

$\Rightarrow \dfrac{1}{10} (x^4+6)^{5/2} + c.$

$\Rightarrow \dfrac{1}{10} (x^4+6)^{5/2} + c$

Q 11 $\int \dfrac{(3x^2+2x+1)\,dx}{(x^3+x^2+x+7)^{1/7}}$

$\Rightarrow \int (x^3+x^2+x+7)^{-1/7} (3x^2+2x+1)\,dx.$

$\Rightarrow \cancel{\int} \dfrac{(x^3+x^2+x)^{6/7}}{6/7} + c$

$\Rightarrow \dfrac{7}{6} (x^3+x^2+x)^{6/7} + c$

8.3

Q1/i $\int 4x^3 (x^4+1)^{5/2} dx$

$\int (x^4+1)^{5/2} 4x^3 dx$

$\Rightarrow \dfrac{(x^4+1)^{7/2}}{7/2} + c$

$= \dfrac{2}{7} (x^4+1)^{7/2} + c$

$\boxed{8.3}$

Q 1 (ii) $\int \dfrac{8x^2}{(x^2+2)^3} dx$

$\Rightarrow \int (x^2+2)^{-3} (8x^2) dx$

$\Rightarrow \int (x^2+2)^{-3} \left(\dfrac{8}{3} \times 3x^2\right) dx$

$\Rightarrow \dfrac{8}{3} \int (x^2+2)^{-3} \cdot 3x^2 dx$

$\Rightarrow \dfrac{8}{3} \left[\dfrac{(x^2+2)^{-2}}{-2}\right] + c$

$\Rightarrow -\dfrac{8}{6} (x^2+2)^{-2} + c$

$\Rightarrow -\dfrac{4}{3} (x^2+2)^{-2} + c$

Q1(iii) $\int \sqrt{1-2x^2} (3x) dx$

$\Rightarrow \int (1-2x^2)^{1/2} (3x) dx$

$$\Rightarrow \int \left[\frac{(1-2x^2)^{1/2+1}}{3/2}\right]+c$$

$$= \boxed{\frac{2}{3}(1-2x^2)^{3/2}+c}$$ Ans

Q 1 (iv) $\int \frac{y+3}{(y^2+6y)^{1/2}}\,dy.$

$$\Rightarrow \int (y^2+6y)^{-1/2}(y+3)\,dy$$

$$\Rightarrow \int (y^2+6y)^{-1/2}\frac{1}{2}(2y+6)\,dy$$

$$\Rightarrow \frac{1}{2}\int (y^2+6y)^{-1/2}(2y+6)\,dy.$$

$$\Rightarrow \frac{1}{2}\left[\frac{(y^2+6y)^{1/2}}{1/2}\right]+c$$

$$\Rightarrow \boxed{(y^2+6y)^{1/2}+c.}$$ Ans

Q 1 (viii) $\int \frac{x^2\,dx}{(1-2x^3)^{2/3}}$

$$\Rightarrow \int (1-2x^3)^{-2/3}x^2\,dx$$

$$\Rightarrow \int (1-2x^3)^{-2/3}\left(\frac{-6}{-6}\right)x^2\,dx.$$

$$\Rightarrow -\frac{1}{6}\int (1-2x^3)^{-2/3}(-6x^2)\,dx$$

$$\Rightarrow -\frac{1}{6}\left[\frac{(1-2x^3)^{1/3}}{1/3}\right]+c$$

$$\Rightarrow \boxed{-\frac{1}{2}(1-2x^3)^{\frac{1}{3}}+c}$$ Ans

Q6 $\int \sqrt{x^2 - 24x^4}\, dx$

$\Rightarrow \int \sqrt{x^2(1-24x^2)}\, dx$

$\Rightarrow \int (1-24x^2)^{1/2} x\, dx$

$\Rightarrow \int (1-24x^2)^{1/2} \frac{-48(x)}{-48}\, dx$

$\Rightarrow \frac{1}{-48}\int (1-24x^2)^{1/2}(-48x)\, dx$

$\Rightarrow \frac{(1-24x^2)^{3/2}}{3/2} + c$

$\Rightarrow \frac{2}{3}(1-24x^2)^{3/2} + c$

Example#1 [224 page]

Q $\int \frac{x\, dx}{\sqrt{4+x^2}} = ?$

Sol

$\Rightarrow \int (4+x^2)^{1/2} x\, dx$

$\Rightarrow \int (4+x^2)^{1/2} \frac{2x}{2}\, dx$

$\Rightarrow \frac{1}{2}\int (4+x^2)^{-1/2}\, 2x\, dx$

$\Rightarrow \frac{1}{2}\left[\frac{(4+x^2)^{1/2}}{3/2}\right] + c$

$\Rightarrow \boxed{\frac{1}{3}(4+x^2)^{1/2} + c}$

Q 6 $\quad$ 8.1 $\quad$ $\int (2x+3)^{2/3}\, dx$. $\quad$ $\boxed{8.1}$ for $\boxed{12}$

Sol

$\Rightarrow \int (2x+3)^{2/3}\, dx$

$\Rightarrow \frac{1}{2}\int 2\cdot(2x+3)^{2/3}\, dx$

$= \frac{3}{5}(2x+3)^{5/3} + c$ Ans

Q 7 $\int (3x+4)^{29}\, dx$

Sol

$\Rightarrow \frac{1}{3}\int 3(3x+4)^{29}\, dx$

$\Rightarrow \frac{1}{3}\left[\frac{(3x+4)^{30}}{30}\right] + c$

$\Rightarrow \frac{1}{90}(3x+4)^{30} + c$

Q 10 $\int \frac{dy}{\sqrt{ay+b}}$.

Sol

$\Rightarrow \int (ay+b)^{-1/2}\, dy$.

$\Rightarrow \frac{1}{a}\int a(ay+b)^{-1/2}\, dy$.

$\Rightarrow \frac{1}{a}\frac{(ay+b)^{1/2}}{1/2} + c$

$\Rightarrow \frac{2}{a}(ay+b) + c$ Ans

Q 18 $\int \frac{dx}{(2x+3)^{2/3}} = ?$

Sol

$\int (2x+3)^{-2/3}\, dx$ d

$\Rightarrow \frac{1}{2}$

$\Rightarrow$

$\Rightarrow \boxed{\frac{3}{}}$

Q 15

Sol

$\Rightarrow \frac{}{3}$

$\Rightarrow \frac{1}{3}$

$\Rightarrow$

$\Rightarrow$

Q 16

Sol

$\Rightarrow$

$$\Rightarrow \frac{1}{2}\int (2x+3)^{-2/3}(2)\,dx$$

8.1

Ex: 2

$$\Rightarrow \frac{1}{2}\left[\frac{(2x+3)^{1/3}}{1/3}\right]+c$$

$$\Rightarrow \boxed{\frac{3}{2}\left[(2x+3)^{1/3}\right]+c}$$

Ans

Q 15 $\int \frac{x^2+3x^{1/2}+4}{3x^{3/4}}\,dx$

Sol

$$\Rightarrow \frac{1}{3}\int\left(\frac{1}{3}\frac{x^2}{x^{3/4}}+\frac{3}{3}\frac{x^{1/2}}{x^{3/4}}+\frac{4}{3}x^{-3/4}\right)dx$$

$$\Rightarrow \frac{1}{3}\int \frac{1}{3}\left(x^{5/4}+3x^{-\frac{1}{4}}+4x^{-3/4}\right)dx$$

$$\Rightarrow \frac{1}{3}\int\left[\frac{x^{9/4}}{9/4}dx+\frac{3x^{3/4}}{3/4}+4\frac{x^{1/4}}{1/4}\right]+c$$

$$\Rightarrow \frac{1}{3}\left[\frac{4}{9}x^{9/4}+4x^{3/4}+16x^{1/4}\right]+c$$

Ans

Q 16 $\int \left(\frac{x+8}{\sqrt{x}}\right)dx = ?$

Sol

$$\int\left(\frac{x+8}{\sqrt{x}}\right).\,x^{-1/2}\,dx$$

$$\Rightarrow \int\left[\frac{x}{x^{1/2}}+8\frac{1}{x^{1/2}}\right]dx$$

$$\Rightarrow \int\left(x^{1/2}+8x^{-1/2}\right)dx$$

$$\Rightarrow \int x^{1/2}\,dx + 8\int x^{-1/2}\,dx$$

$$\Rightarrow \frac{x^{3/2}}{3/2} + 8\,\frac{x^{1/2}}{1/2} + c$$

$$\Rightarrow \frac{2}{3}x^{3/2} + 16\,x^{1/2} + c.$$

$$\Rightarrow \boxed{\frac{2}{3}x^{3/2} + 16x^{1/2} + c.}$$ Ans

Q $$\int \frac{(x^2+2x+4)^{+1/3}}{(x+1)}\,dx$$

Sol

$$\Rightarrow \int \frac{(x+1)}{(x^2+2x+4)^{-1/3}}\,dx$$

$$\Rightarrow \int (x^2+2x+4)^{1/3}\,(x+1)\,dx$$

$$\Rightarrow \int (x^2+2x+4)^{1/3}\,(x+1)\,dx$$

$$\Rightarrow \frac{1}{2}\int (x^2+2x+4)^{1/3}\,(2(x+1))\,dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{(x^2+2x+4)^{4/3}}{4/3}\right] + c$$

$$\Rightarrow \boxed{\frac{3}{8}(x^2+2x+4)^{4/3} + c}$$ Ans

Integral

Definite (With limit)

Indefinite (Without limit).

$$\int_{\boxed{a}}^{\boxed{b}} f(x)\,dx = \text{upperlimit} - \text{lowerlimit}$$

$$= f(b) - f(a)$$

$\boxed{b} \rightarrow$ upper limit

$\boxed{a} \rightarrow$ lower limit

QWN

Q $\int_1^2 2x\,dx$

Sol

$\Rightarrow 2\int_1^2 x\,dx$

$\Rightarrow \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_1^2$

$\Rightarrow \left[x^2\right]_1^2$

$\Rightarrow (2)^2 - (1)^2$

$\Rightarrow 4 - 1$

$\Rightarrow 3$ Ans

$\boxed{8.1}$

(19) $\int_{-1}^{1} (2x^2+4)^3 (4x)\, dx$

Sol

$\Rightarrow \frac{1}{4} \int_{-1}^{1} \left[\frac{(2x^2+4)^4}{4} \right]_{-1}^{+1}$

$\Rightarrow \frac{1}{4} \left\{ (2(1)^2+4)^4 - (2(-1)^2+4)^4 \right\}$

$\Rightarrow \frac{1}{4} \left\{ (2+4)^4 - (2+4)^4 \right\}$

$\Rightarrow \frac{1}{4} \left\{ \cancel{(6)^4} - \cancel{(6)^4} \right\}$

$\Rightarrow 0$ Ans

1

(20) $\int_0^2 (x^2+bx+c)^{-2/3} \left(x+\frac{b}{2}\right) dx$

Sol

$\Rightarrow \int_0^2 (x^2+bx+c)^{-2/3} \left(\frac{2x+b}{2}\right) dx$

$\Rightarrow \frac{1}{2} \int_0^2 (x^2+bx+c)^{-2/3} \left(\frac{2x+b}{2}\right) dx$

$\Rightarrow \frac{1}{2} \left[\frac{(x^2+bx+c)^{1/3}}{1/3} \right]_0^2$

$\Rightarrow \frac{3}{2} \left[(x^2+bx+c)^{1/3} \right]_0^2$

$$\Rightarrow \frac{3}{2}\left[\{(2)^2+b\cdot 2+c\}^{1/3}-\{0^2+0\cdot b+c\}^{1/3}\right]$$

$$\Rightarrow \boxed{\frac{3}{2}\left[\{4+2b+c\}^{1/3}-c^{1/3}\right]}$$ Ans

Q(14) $\displaystyle\int_0^2 \frac{dx}{\sqrt{1+x}+\sqrt{x}}$

$\Rightarrow$

$$\Rightarrow \int_0^2 \left[\frac{dx}{\sqrt{1+x}+\sqrt{x}} \times \frac{\sqrt{1+x}-\sqrt{x}}{\sqrt{1+x}-\sqrt{x}}\right] dx$$

$$\Rightarrow \int_0^2 \frac{(\sqrt{1+x}-\sqrt{x})}{(\sqrt{1+x})^2-(\sqrt{x})^2}\, dx$$

$$\Rightarrow \int_0^2 \left(\frac{\sqrt{1+x}-\sqrt{x}}{1+x-x}\right) dx$$

$$\Rightarrow \int_0^2 (\sqrt{1+x}-\sqrt{x})\, dx$$

$$\Rightarrow \int_0^2 \sqrt{1+x}\, dx - \int_0^2 \sqrt{x}\, dx$$

$$\Rightarrow \frac{2}{3}\left[(1+x)^{3/2}\right]_0^2 - \frac{2}{3}\left[x^{3/2}\right]_0^2$$

$$\Rightarrow \frac{2}{3}\left\{(1+2)^{3/2}-(1+0)^{3/2}\right\} - \frac{2}{3}\left\{2^{3/2}-0^{3/2}\right\}.$$

$3^1 \cdot 3^{1/2} \Rightarrow 3^{3/2}$

Q.4 1 → 7

Q.5 → 7, 9, 13

$\Rightarrow \frac{2}{3}\left\{3^{3/2} - 1^{3/2}\right\} - \frac{2}{3}\left\{2^{3/2}\right\}.$

$\cancel{\Rightarrow \frac{2}{3} \cdot 3^{3/2} - \frac{2}{3} - \frac{2^{1+3/2}}{3}}$

$\cancel{\Rightarrow 2 \cdot 3^{3/2-1} - \frac{2}{3} - \frac{2^{5/2}}{3}}.$

$\cancel{\Rightarrow 2 \cdot 3^{1/2} - \frac{2}{3} - \frac{2^{5/2}}{3}}.$

$\Rightarrow \frac{2}{3}\left[3^{3/2} - 1^{3/2} - 2^{3/2}\right]$

$\Rightarrow \frac{2}{3}\left[3\sqrt{3} - 1 - 2\sqrt{2}.\right]$

$\boxed{B\text{-}4}$

Q3 $\int_{-3}^{-1} \frac{dx}{(x-1)^3}$ Q.4

Sol.

$\Rightarrow \int_{-3}^{-1} (x-1)^{-3}\, dx$

$\Rightarrow \left[\frac{(x-1)^{-2}}{-2}\right]_{-3}^{-1}$

$\Rightarrow -\frac{1}{2}\left[(x-1)^{-2}\right]_{-3}^{-1}$

$\Rightarrow$

$\Rightarrow -\frac{1}{2}\left\{(-1-1)^{-2} - (-3-1)^{-2}\right\}$

$\Rightarrow -\frac{1}{2}\left\{(-2)^{-2} - (-4)^{-2}\right\} = -\boxed{\frac{3}{32}}$ A

E.3

Algebraic substitution

Q1(i). $\int 4x^3 (x^4+1)^{1/2}\, dx$

Sol: Put $x^4+1 = t$

$4x^3\, dx = dt$

$\Rightarrow \int (t)^{3/2}\, dt$

$\Rightarrow \left[\dfrac{t^{5/2}}{5/2}\right] + c$

$\Rightarrow \dfrac{2}{5}\, t^{5/2} + c$

$\Rightarrow \dfrac{2}{5} (x^4+1)^{5/2} + c.$

Q1:- (xii) $\int \dfrac{x}{\sqrt{1+x}}\, dx$

Sol: $\Rightarrow \int (1+x)^{-1/2} x\, dx$

Put $1+x = t$ or $x = t-1$

$dx = dt$

$\Rightarrow \int (t)^{-1/2} (t-1)\, dt$

$\Rightarrow \int (t^{1/2} - t^{-1/2})\, dt$

$$\Rightarrow \frac{t^{3/2}}{3/2} - \frac{t^{1/2}}{1/2} + c$$

$$\Rightarrow \frac{2}{3} t^{3/2} - 2t^{1/2} + c$$

$$\Rightarrow \frac{2}{3}(1+x)^{3/2} - 2(1+x)^{1/2} + c$$

Q 1 (xiii) $\int (3x+2)(x-1)^{-1/2} dx$

Sol

$$\Rightarrow \int (x-1)^{-1/2}(3x+2)\,dx$$

$\Rightarrow$ put $x-1 = t \Rightarrow x = t+1$

$dx = dt$

$$\Rightarrow \int (t)^{-1/2}(3(t+1)+2)\,dt$$

$$\Rightarrow \int (t)^{-1/2}(3t+3+2)\,dt$$

$$\Rightarrow \int (t)^{-1/2}(3t+5)\,dt$$

$$\Rightarrow \int (3t^{1/2} + 5t^{-1/2})\,dt$$

$$\Rightarrow 3\int t^{1/2}dt + 5\int t^{-1/2}dt$$

$$\Rightarrow 3\left[\frac{t^{3/2}}{3/2}\right] + 5\left[\frac{t^{1/2}}{1/2}\right] + c$$

$$\Rightarrow \frac{\cancel{6}\,2}{3} t^{3/2} + 10\, t^{1/2} + c$$

$\Rightarrow 2(x-1)^{3/2} + 10\,(x-1)^{1/2} + c$

Q 1 (xiii) $\int x^2 \sqrt{4+x}\, dx$

Sol

$\Rightarrow \int \sqrt{4+x}\; x^2\, dx$

put $4+x = t \Rightarrow x = t-4$

$dt = dx$

S $\Rightarrow \int (t)^{1/2} \cdot (t-4)^2\, dt$

$\Rightarrow \int t^{1/2} (t^2 - 8t + 16)\, dt$

$\Rightarrow \int (t^{5/2} - 8t^{3/2} - 16t^{1/2})\, dt$

$\rightarrow \int t^{5/2} dt - 8\int t^{3/2} dt - 16\int t^{1/2} dt$

$\Rightarrow \frac{2}{7} t^{7/2} - 8\,\frac{t^{5/2}}{5/2} - 16\,\frac{t^{3/2}}{3/2} + c$

$\rightarrow \frac{2}{7}(4+x)^{7/2} - \frac{16}{5}(x+4)^{5/2} - \frac{32}{3}(4+x)^{3/2} + c$

Q 1 (xiv) $\int (2x^2-3)^{1/3}\, x^3\, dx$

Sol

$\Rightarrow \int (2x^2-3)^{1/3}\, x^2 \cdot x\, dx$

$\Rightarrow$ ... $= t \Rightarrow x^2 = \frac{t+3}{2}$

... dt

... t

... 4

... $\frac{t+3}{2}\Big)\, \frac{dt}{4}$

$y = \ln \sin x$
$\frac{dy}{dx} = \frac{1}{\sin x} \frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$

16. $\int (x^2 - 2x + 1)^{4/3} dx$

$\Rightarrow \int \left[(x-1)^2\right]^{4/3} dx$

$\Rightarrow \int (x-1)^{8/3} dx$

$\Rightarrow \left[\frac{(x-1)^{11/3}}{11/3}\right] + c$

$\Rightarrow \frac{3}{11} (x-1)^{11/3} + c$

Form IV

$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$

Example:

(1) $\int \frac{1}{x} dx = \ln x + c$

(2) $\int \frac{\cos x}{\sin x} dx = \ln(\sin x) + c$

E. B.

Q 1 (vii) $\int \frac{x\, dx}{3x^2 - 4}$ Q 9, 10.

Sol

$\Rightarrow \frac{1}{6} \int \frac{6x\, dx}{3x^2 - 4}$

$\ln a^x = x \ln a$ 7

$\Rightarrow \frac{1}{6} \ln(3x^2-4)+c$

$\Rightarrow \ln(3x^2-4)^{1/6}+c$

8.3 = 10, 9
8.4 = 1 → 7

own

Q $\int \frac{2x+3}{x^2+3x}\,dx$

$\Rightarrow \ln(x^2+3x)+c.$

own

Q $\int \frac{x+1}{x^2+2x}\,dx$

Sol

$= \frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx$

$\because \int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$

$\Rightarrow \frac{1}{2}\ln(x^2+2x)+c$

$\Rightarrow \ln(x^2+2x)^{1/2}+c$

$\Rightarrow \ln\sqrt{x^2+2x}+c$

8.3

Q 1 (iv) $\int \frac{x+2}{x-3}\,dx = ?$

Sol

$\Rightarrow \int \frac{x+2}{x-3}\,dx$

$x-3 \overline{) x+ }$

$Q + \frac{R}{D}$

Quient + $\frac{\text{Ramider}}{\text{divisor}}$

$2\overline{)5}$ $\frac{5}{2}$ $2 + \frac{1}{2}$

$$\begin{array}{r} 1 \\ x-3 \overline{) x+2} \\ -(x-3) \\ \hline 5 \end{array}$$

$\Rightarrow \int \left(1 + \frac{5}{x-3}\right) dx$

$\Rightarrow \int dx + 5 \int \frac{1}{x+3} dx$

$\Rightarrow x + 5\ln(x-3) + c.$

$\Rightarrow x + \ln(x-3)^5 + c$

$$\begin{array}{r} 1 \\ x-3 \overline{) x+7} \\ x-3 \\ \hline 10. \end{array}$$

Q $\int \frac{x+7}{x-3} dx.$

$\Rightarrow \int \left(1 + \frac{10}{x-3}\right) dx$

$\Rightarrow \int dx + 10 \int \frac{1}{x-3} dx$

$\Rightarrow x + 10 \ln(x-3) + c$

$\Rightarrow x + \ln(x-3)^{10} + c$

8.13
$1 \rightarrow 16$
complete [illegible]

8) $\int \frac{x+2}{x-3}\,dx$

Sol:

$\Rightarrow \int \frac{x+2}{x-3}\,dx$ $\qquad x-3 \,\big)\, x+2 \,\big(\, 1$; $\; x-3$; $\; 5$

$\Rightarrow \int \left(1 + \frac{5}{x-3}\right) dx$

$\Rightarrow \int dx + 5\int \frac{1}{x-3}\,dx$

$\Rightarrow x + 5\ln(x-3) + c$

$\Rightarrow x + \ln(x-3)^5 + c$ Ans

$\sin x + c$	$\int \cos x\,dx$
$-\cos x + c$	$\int \sin x\,dx$
$\tan x + c$	$\int \sec^2 dx$
$\sec x + c$	$\int \sec x \tan x\,dx$
$\ln \sin x$	$\int \cot x\,dx$
$-\cot x + c$	$\int \operatorname{cosec}^2 x\,dx$
$-\operatorname{cosec} x + c$	$\int \operatorname{cosec} x \cot x\,dx$

$\ln x + c \;\Big|\; \int \frac{1}{x}$

Q.3 Q 3 (vi).

Q $\int \tan x \, dx$ $\qquad (\cos x^{-1}) \neq \cos^{-1}$ ⇓ Inverse trigonometric function

Sol

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

$\Rightarrow -1 \int \frac{-\sin x}{\cos x} dx$

$\Rightarrow (-1) \ln (\cos x) + c.$

$\Rightarrow \ln (\cos x)^{-1} + c.$

$\Rightarrow \ln \left(\frac{1}{\cos x} \right) + c$

$\Rightarrow \ln \sec x + c.$

Note:- اگر trigonometric single function ہو اور اس کے ساتھ angle (θ) کو suppose کریں گے۔

Ex/ $\int \sin (3x + 2) \, dx$ $\qquad$ let $u = 3x + 2$

اگر ایک سے زیادہ Tri.Func کو suppose کریں تو اس کا Derivative آگے بیچ میں نظر آ رہا ہو

Ex/ $\int \sin^2 x \cdot \cos x \, dx$ $\qquad$ Let $\sin x = u$

Proof:- $\int \sin(3x+2)\,dx = ?$

Sol:

put $3x+2 = t \Rightarrow x = \frac{t-2}{3}$

$3dx = dt$

$dx = \frac{dt}{3}$

$\Rightarrow \int \sin t \frac{dt}{3}$

$\Rightarrow \frac{1}{3}\int \sin t\,dt.$

$\Rightarrow \frac{1}{3}(-\cos t) + c.$

$\Rightarrow -\frac{\cos(3x+2)}{3} + c$

Q 11 $\int \cos 2y\,dy$

Sol:

$\Rightarrow \int \cos 2y\,dy$

$\Rightarrow$ put $2y = t$

$2dy = dt$

$dy = \frac{dt}{2}$

... dt

... $dt.$

... $t + c$

$\Rightarrow \frac{1}{2} \sin t + c$

$\Rightarrow \frac{1}{2} \sin 2y + c.$

Q 3 (xiii)

Q $\int \sec^2 2ax \, dx \Rightarrow ?$

Sol:

$\Rightarrow$ put $2ax = t \Rightarrow \frac{t}{2a} = x$

$2a\,dx = dt$

$dx = \frac{dt}{2a}$

$\Rightarrow \int \sec^2 t \frac{dt}{2a}$

$\Rightarrow \frac{1}{2a} \int \sec^2 t \, dt.$

$\Rightarrow \frac{1}{2a} \tan t + c.$

$\Rightarrow \frac{1}{2a} \tan 2ax + c.$

Q3. (viii) $\int \frac{\sin x + \cos x}{\cos x} dx$

Sol:

$\int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} \right) dx$

$\Rightarrow \int \frac{\sin x}{\cos x} dx + \int dx$

$\Rightarrow \int \tan x \, dx + x + c.$

$\Rightarrow \ln(\sec x) + x + c$

Q. 3 $\int (1+\tan x)^2 dx$ $\because \{1+\tan^2 x = \sec^2 x\}$

Sol:

$\Rightarrow \int (1+\tan^2 x + 2\tan x) dx$

$\Rightarrow \int (\sec^2 x + 2\tan x) dx$

$\rightarrow \int \sec^2 x \, dx + 2\int \tan x \, dx$

$\Rightarrow \cancel{\sec x} \tan x + 2\ln \sec x + c$

Q 3 (xiv) $\int \frac{\sec x \tan x}{a+b\sec x} dx$

Sol

$\Rightarrow \int \frac{\sec x \tan x}{a+b\sec x} dx$

$\Rightarrow \frac{1}{b} \int \frac{b\sec x \tan x}{a+b\sec x} dx$

$\Rightarrow \frac{1}{b} \ln(a+b\sec x) + c$

$\Rightarrow \ln(a+b\sec x)^b + c.$

Q 3 (xvi) $\int \frac{\sin \ln x}{x(3-\cos\ln x)^{1/2}} dx$

$\Rightarrow \int (3-\cos\ln x)^{-1/2} \left(\frac{\sin \ln x}{x}\right) dx$

$\Rightarrow 2(3-\cos\ln x)^{1/2} + c$

$3 - \cos \ln x = y$

$0 - (-\sin \ln x) \frac{d}{dx} \ln x = \frac{dy}{dx}$

$\frac{\sin \ln x}{x} = \frac{dy}{dx}$

8.3

Q 3 (iii) $\int \sin^2 x \cos x \, dx$

Sol

Let $\sin x = u$

$\cos x \, dx = du$

$dx = \frac{du}{\cos x}$

$\Rightarrow \int u^2 . \cos x \frac{du}{\cos x}$

$\Rightarrow \int u^2 \, du$

$\Rightarrow \int \frac{u^3}{3} + c$

$\Rightarrow \frac{(\sin x)^3}{3} + c$

Q 3 (v) $\int x . \cos x^2 \, dx$.

Sol

put $x^2 = u$

$2x \, dx = du$

$x \, dx = \frac{du}{2}$

$\Rightarrow \int \cos u \frac{du}{2}$

$\Rightarrow \frac{1}{2} \int \cos u \, du$

$= \frac{1}{2} \sin u + c.$

$\Rightarrow \frac{1}{2} \sin x^2 + c$

Q3(x) $\int e^x \cos e^x \, dx$

Put $e^x = u$

$dx e^x = du$

$\Rightarrow \int \cos u \, du$

$\Rightarrow \sin u + c$

$\Rightarrow \sin e^x + c$

(1) $\int x \cos x^2 \, dx$

$\int e^x \tan e^x \, dx$

$\int \cot(5x - 9) \, dx$

$\int x^2 \sin x^3 \, dx$

$\int x^4 \tan x^5 \, dx$

$\int (1+x^2)^{3/5} \, x \, dx$

$\int \tan(3x-3) \, dx$

$\int \sec^2(4x+13) \, dx$

$\int e^x \sec^2 e^x \, dx$

(10) $\int \tan^2 x \sec x \, dx$

$\boxed{8.4}$

Q 1. $\int_0^3 \sqrt[3]{(3t-1)^2}\,dt.$

Sol

$\Rightarrow \int_0^3 \sqrt[3]{(3t-1)^2}\,dt.$ $\quad \sqrt[3]{8^5} = 16$

$\Rightarrow \frac{1}{3}\int_0^3 (3t-1)^{2/3}\,3\,dt.$

$\Rightarrow \int_0^3 \frac{1}{3}\left[\frac{(3t-1)^{5/3}}{5/3}\right]_0^3$

$\Rightarrow \frac{1}{5}\left[(3t-1)^{5/3}\right]_0^3.$ $\quad 8^3 \quad (2^3)^{5/3} \quad 2^5$

$\Rightarrow \frac{1}{5}\left[(3(3)-1)^{5/3} - (3(0)-1)^{5/3}\right]$

$\Rightarrow \frac{1}{5}\left\{8^{5/3} - (-1)^{5/3}\right\}.$

$\Rightarrow \frac{1}{5}\left\{\sqrt[3]{(8)^{5/3}} - (-1)^{5/3}\right\}.$

$\Rightarrow \frac{1}{5}\left\{2^5 + 1\right\}.$

$\Rightarrow \frac{1}{5}(32+1) \Rightarrow \frac{33}{5}$

Q 2. $\int_{-2}^{1} \sqrt{2-x}\,dx$

Sol

$\int_{-2}^{1} \sqrt{2-x}\,dx$

8.4

$$\Rightarrow -1\int_{-2}^{1}\sqrt{2-x}\left(\frac{-1}{-1}\right)dx$$

$$\Rightarrow -1\int_{-2}^{1}\sqrt{2-x}\,(-1)\,dx$$

$$\Rightarrow -1\left[\frac{(2-x)^{3/2}}{3/2}\right]_{-2}^{1}$$

$$\Rightarrow -2/3\left[(2-x)^{3/2}\right]_{-2}^{1}$$

$$\Rightarrow -\frac{2}{3}\left\{(2-1)^{3/2}-(2+2)^{3/2}\right\}$$

$$\Rightarrow -\frac{2}{3}\left\{1^{3/2}-4^{3/2}\right\}$$

$$\Rightarrow -\frac{2}{3}\left\{1-8\right\}$$

$$\Rightarrow -\frac{2}{3}(-7)=\frac{14}{3}$$ Ans

Q3 $\int\frac{dx}{(x-1)^3}$

Sol

done

Q4 $\int_{-2}^{1}x\sqrt{2x^2+3}\,dx$

Sol

$$\int_{-2}^{1}\sqrt{2x^2+3}\cdot x\,dx$$

$$\Rightarrow \frac{1}{4}\int_{-2}^{1}(2x^2+3)^{1/2}\cdot 4x\,dx$$

$$\Rightarrow \frac{1}{4}\left[\frac{(2x^2+3)^{1/2+1}}{3/2}\right]_{-2}^{1}$$

$$\Rightarrow \frac{1}{4_2}\times\frac{2}{3}\left\{\{2(1)^2+3\}^{3/2}-\{2(-2)^2+3\}^{3/2}\right\}$$

$$\Rightarrow \frac{1}{6}\left\{5^{3/2}-11^{3/2}\right\}.$$

$$\Rightarrow \frac{1}{6}\left[5\sqrt{5}-11\sqrt{11}\right]$$

Ans

85 $\int_0^5 \frac{2x+3}{\sqrt{x^2+3x+1}}\,dx$

Sol

$$\int_0^5 \frac{2x+3}{\sqrt{x^2+3x+1}}\,dx$$

$$\Rightarrow \int_0^5 (x^2+3x+1)^{-1/2}(2x+3)\,dx$$

$$\Rightarrow \int_0^5 \left[\frac{(x^2+3x+1)^{1/2}}{1/2}\right]_0^5$$

$$\Rightarrow 2\left[(x^2+3x+1)^{1/2}\right]_0^5$$

$$\Rightarrow 2\left[\{(5)^2+3(5)+1\}^{1/2}-\{0+0+1\}^{1/2}\right].$$

$$\Rightarrow 2\left[\sqrt{41}-1\right\}.$$

Ans

86 $\int_1^2 (2x+1)^3\sqrt{x^2+x+1}\,dx$

Sol

$$\Rightarrow \int_1^2 (2x+1)^3\sqrt{x^2+x+1}\,dx$$

8.11

Q Solve the differential equations.

Q A equation contain the derivative of the function $f(x)$ is called differential equation

Ex ① $\frac{dy}{dx} = \cos x$

② $\frac{dy}{dx} = 2x^2+5$

③ $\frac{dy}{dx} = \cos x$

Variable Separate form.

Q3 $\frac{dy}{dx} = x + \sin x$

$y = 3$ when $x = 0$ condition

Sol

$dy = (x + \sin x)dx$

Now we integrating bothsides

$\int dy = \int (x + \sin x)dx$

$\int dy = \int x\,dx + \int \sin x\,dx$

$y = \frac{x^2}{2} - \cos x + c$

Put $y = 3$; $x = 0$.

$3 = 0 - \cos 0^\circ + c$

$3 + 1 = c$

$c = \cancel{2}4$

Now, Solution become

$$y = \frac{x^2}{2} - \cos x + 4$$

Note:- No condition

Q 8 $\frac{ds}{dt} = s^2 t \sqrt{1+t^2}.$

$\frac{ds}{s^2} = dt\, t\sqrt{1+t^2}$

$s^{-2} ds = t\sqrt{1+t^2}\, dt$

On integrating bothsides

$\int s^{-2} ds = \int t\sqrt{1+t^2}\, dt$

$\frac{s^{-1}}{-1} = \frac{1}{2} \frac{2}{3} (1+t^2)^{2/3} dt.$

$-\frac{1}{s} = \frac{1}{3} (1+t^2)^{2/3} + c$

Q 9 $\frac{du}{dx} = x^3 \sqrt{u}$

$\frac{du}{\sqrt{u}} = x^3 dx$

On integrating bothsides

$\int u^{-1/2} du = \int x^3 dx$

$2 u^{1/2} = \frac{x^4}{4} + c$

Q 10 $\frac{dv}{dt} = \sqrt{5v - 7}\,(t \cos t^2)$

$$\frac{d\nu}{\sqrt{5\nu-7}} = (t\cos t^2)\,dt$$

On integrating both sides.

$$\int (5\nu-7)^{-1/2}\,d\nu = \int (t\cos t^2)\,dt$$

$$\frac{(5\nu-7)^{1/2}}{\frac{5}{2}} = \frac{1}{2}\int t\cos t^2\, 2t\,dt$$

$$\frac{2}{5}(5\nu-7)^{1/2} = \frac{1}{2}\sin t^2 + c$$

Q 11 $\frac{dy}{dx} = \sin^2 y \cdot \cos^2 x \sin x$

$$\frac{dy}{\sin^2 y} = \cos^2 x \sin x\, dx$$

On integrating both sides.

$$\int \frac{dy}{\sin^2 y} = \int \cos^2 x \sin x\, dx$$

$$\int \frac{1}{\sin^2 y}\cdot dy = -\int \cos^2 x\,(-\sin x)\,dx$$

$$\int \operatorname{cosec}^2 y\, dy = -\int \cos^2 x\,(-\sin x)\,dx$$

$$-\cot y = -\frac{\cos^3 x}{3} + c$$

$$\cot y = \frac{\cos^3 x}{3} + c$$

Q13 $\frac{dy}{dx} = \frac{\sin^2 y}{\cos^2 x}$

Sol $\frac{dy}{\sin^2 y} = \frac{\cos^2 x\, dx}{\cos^2 x}$

$\operatorname{cosec}^2 y\, dy = \sec^2 x\, dx$

On integrating both sides

$\int \operatorname{cosec}^2 y\, dy = \int \sec^2 x\, dx$

$-\cot y = \tan x + c$

$0 = \tan x + \cot y + c$

Q14 $x^2 \frac{dy}{dx} = x^4 y^2 + y^2$

Sol $\frac{dy}{y^2} = \frac{(x^4+1)}{x^2} dx$

$y^{-2} dy = (x^2 + x^{-2}) dx$

On integrating both sides.

$\int y^{-2} dy = \int (x^2 + x^{-2}) dx$

$\frac{y^{-1}}{(-1)} = \frac{x^3}{3} - \frac{x^{-1}}{1} + c$

$-\frac{1}{y} = \frac{x^3}{3} - \frac{1}{x} + c$

$0 = \frac{1}{y} + \frac{x^3}{3} - \frac{1}{x} + c$

Q15 $y(4+x^2)\frac{dy}{dx} = x(1+y^2)^2$

Sol $\frac{y\, dy}{(1+y^2)^2} = \frac{x}{(1+x^2)} dx$

On integrating

$$\int (1+y^2)^{-2}\, y\, dy = \int \frac{x}{1+x^2}\, dx$$

$$\frac{1}{2}\int (1+y^2)^{-2}\, 2y\, dy = \frac{1}{2}\ln(1+x^2)+c$$

$$\frac{1}{2}\int (1+y^2)^{-2}(2y)\, dy = \frac{1}{2}\ln(1+x^2)+c$$

$$\frac{1}{2}\frac{(1+y^2)^{-1}}{-1} = \frac{1}{2}\ln(1+x^2)+c$$

$$-\frac{1}{2}(1+y^2) = \ln\sqrt{1+x^2}+c$$

Q Ex $\frac{dy}{dx} = x^2y^3$; $y=-1$; $x=1$

Sol:

$$\frac{dy}{dx} = x^2y^3$$

$$\frac{dy}{y^3} = \int x^2\, dx$$

On integ

$$\int y^{-3}\, dy = \int x^2\, dx$$

$$\frac{y^{-2}}{(-2)} = \frac{x^3}{3}+c$$

$$-\frac{1}{2y^2} = \frac{x^3}{3}+c$$

Put $x=1$; $y=-1$

$$-\frac{1}{2(-1)^2} = \frac{(1)^2}{3}+c$$

$$-\frac{1}{2} = \frac{1}{3}+c$$

$$-\frac{1}{2}-\frac{1}{3} = c$$

$-\frac{15}{6} = c$

$\therefore$ Req. solution will be

$$-\frac{1}{2y^2} = \frac{1x^3}{3} - \frac{5}{6}$$

8 Q9 $\frac{ds}{dt} = \sqrt{s+1}\sqrt{3t+1}$

$s=3;\ t=5$

Sol

$$\frac{ds}{\sqrt{s+1}} = \sqrt{3t+1}\,dt$$

On integ b/s.

$$\int \frac{ds}{\sqrt{s+1}} = \int \sqrt{3t+1}\,dt$$

$$\frac{\sqrt{s+1}}{1/2} = \frac{1}{3}\frac{(3t+1)^{3/2}}{3/2} + c$$

$$2\sqrt{s+1} = \frac{2}{9}(3t+1)^{3/2} + c$$

Put $s=3$; $t=5$

$$2\sqrt{3+1} = \frac{2}{9}(3\cdot5+1)^{3/2} + c$$

$$2\times2 = \frac{2}{9}(16)^{3/2} + c$$

$$4 = \frac{2}{9}(64) + c$$

$$4 - \frac{2}{9}(64) = c$$

$$c = \frac{36 - 128}{9}$$

$$c = -9\frac{2}{9}$$

$\therefore$ Req Solution will be

$$2\sqrt{s+1} = \frac{2}{3}(3t+1)^{3/2} - \frac{92}{9}$$

Q20 $\frac{dr}{ds} = \frac{\sqrt{r^2+1}\ \sqrt{2s+3}}{r}$ $r(11)=\sqrt{3}$

Sol

$$\frac{r\,dr}{\sqrt{r^2+1}} = ds\sqrt{2s+3}.$$

On integrating b/sides.

$$\int \frac{r}{\sqrt{r^2+1}}\,dr = \int \sqrt{2s+3}\,ds.$$

$$\frac{1}{2}\int (r^2+1)^{-1/2}\,2r\,dr = \frac{1}{2}\cdot\frac{2}{3}(2s+3)^{3/2}+c$$

$$\frac{1}{2}\frac{(r^2+1)^{1/2}}{1/2} = \frac{1}{3}(2s+3)^{3/2}+c.$$

$$(r^2+1)^{1/2} = \frac{1}{3}(2s+3)^{3/2}+c$$

Put $r=\sqrt{3}$
$s=11$

$$(3+1)^{1/2} = \frac{1}{3}(22+3)^{3/2}+c$$

$$2 = \frac{1}{3}\,25\sqrt{25}+c$$

$$2-\frac{1}{3}(125) = c$$

$$c = \frac{6-125}{3}$$

$$c = -\frac{119}{3}$$

$$c = -\frac{119}{3}$$

$\therefore$ Req solution is

$$(r^2+1)^{1/2} = \frac{1}{3}(2s+3)^{3/2} - \frac{119}{3}$$

Q 21 $\frac{dy}{dx} = \frac{\sqrt{1+\cos y}}{\sin y}$

Sol

$y(3) = \frac{\pi}{2}$

$dy \sin y = \sqrt{1+\cos y}\, dx$

On integrating

$\Rightarrow \int \frac{\sin y}{\sqrt{1+\cos x}} dy = \int dx$

$\Rightarrow \int \frac{\sin y}{\sqrt{1+\cos x}} dy = \int dx$

$\Rightarrow -\int (1+\cos y)^{-1/2} (-\sin y) dy = \int dx$

$\Rightarrow -\frac{(1+\cos y)^{1/2}}{1/2} = x + c$

$\Rightarrow -2(1+\cos y)^{1/2} = x + c.$

Put $y = \pi/2$
$x = 3$

$\therefore -2(1 + \cos \pi/2)^{1/2} = 3 + c$

$-2(1+0)^{1/2} = 3 + c$

$-2 - 3 = c$

$\Rightarrow c = -5$

$\therefore$ Req. solution will be

$-2(1+\cos y)^{1/2} = x - 5$

$\sqrt{1+\cos y} = -\frac{x}{2} + \frac{5}{2}$

3

Q 22 $y\frac{dy}{dx} = x(y^4+2y^2+1)$; $y(-3)=1$

Sol:

$$y\frac{dy}{dx} = x(y^4+2y^2+1)$$

$$\frac{y\,dy}{(y^4+2y^2+1)} = x\,dx$$

On integrating both sides

$$\int \frac{y}{(y^2+1)^2}\,dy = \int x\,dx$$

$$\frac{1}{2}\int (y^2+1)^{-2}(2y)\,dy = \frac{x^2}{2}+c$$

$$\frac{(y^2+1)^{-1}}{-1} = \frac{x^2}{2}+c$$

$$-\frac{1}{2(1+y^2)^2} = \frac{x^2}{2}+c$$

Put $y=1$; $x=-3$

$$-\frac{1}{2(1+1)} = \frac{9}{2}+c$$

$$-\frac{1}{4} = \frac{9}{2}+c$$

$$-\frac{1}{4}-\frac{9}{2} = c$$

$$\frac{-1-18}{4} = c$$

$$C = -\frac{19}{4}$$

∴ Req solution will be

$$-\frac{1}{2(y^2+1)} = \frac{-x^2}{2} + \frac{19}{4}$$

Q 24 Imp $\frac{dw}{dz} = \sqrt{wz - 2w - 3z + 6}$

$w = 12$

$z = 6$

Sol

$$\frac{dw}{dz} = \sqrt{w(z-2) - 3(z-2)}$$

$$\frac{dw}{dz} = \sqrt{(w-3)(z-2)}$$

$$\frac{dw}{\sqrt{w-3}} = (z-2)^{1/2} dz$$

On integrating bothsides

$$\int \frac{dw}{\sqrt{w-3}} = \int (z-2)^{1/2} dz$$

$$\int (w-3)^{-1/2} dw = \int (z-2)^{1/2} dz$$

$$2(w-3)^{1/2} = \frac{2}{3}(z-2)^{3/2} + c$$

Put $w = 12$

$z = 6$

$$\therefore 2(12-3)^{1/2} = \frac{2}{3}(6-2)^{3/2} + c$$

$$2 \times 3 = \frac{2}{3} 4\sqrt{4} + c$$

$$6 - \frac{16}{3} = c$$

$$c = \frac{2}{3}$$

$\therefore$ Req solution will be

$$2\sqrt{w-3} = \frac{2}{3}(z-2)^{3/2} + \frac{2}{3}$$

$$\sqrt{w-3} = \frac{(z-2)^{3/2}}{3} + \frac{1}{3}$$

$$\sqrt{w-3} = \frac{1}{3}\left\{(z-2)^{3/2} + 1\right\}$$

Q 25 $2 + 2y\frac{dy}{dx} = 1 + 3x^2$; $y(2) = 1$

Sol

$$2(1+y)\frac{dy}{dx} = 1 + 3x^2 \cancel{+2}$$

$$2y\frac{dy}{dx} = 3x^2 - 1$$

$$2y\,dy = (3x^2 - 1)\,dx$$

On integrating bothsides.

$$2\int y\,dy = \int (3x^2 - 1)\,dx$$

$$\frac{2y^2}{2} = \frac{3x^3}{3} - x + c$$

$$y^2 = x^3 - x + c$$

Put $x = 2$; $y = 1$

$$\therefore\ 1^2 = 2^3 - 2 + c$$

$$1 + 2 - 8 = c$$

$$c = -5$$

$\therefore$ The req solution will be

$$y^2 = x^3 - x - 5$$

Q.5

Q 9 $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

$\Rightarrow \int_0^1 \frac{dx}{\sqrt{(2)^2-x^2}}$

Put $x = 2\sin\theta$ when $x=0$

$dx = 2\cos\theta\, d\theta$ $\quad 0 = 2\sin\theta$

$\theta = 0$

$x = 1$

$1 = 2\sin\theta$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \pi/6.$

$\Rightarrow \int_0^{\pi/6} \frac{2\cos\theta\, d\theta}{\sqrt{4-4\sin^2\theta}}$

$\Rightarrow \int_0^{\pi/6} \frac{2\cos\theta\, d\theta}{2\cos\theta}$

$\Rightarrow \int_0^{\pi/6} d\theta$

$\Rightarrow \left[\theta\right]_0^{\pi/6}$

$\Rightarrow \left[\pi/6 - 0\right]$

$\Rightarrow \left[\pi/6.\right]$

Q9 $\int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{x^2\sqrt{x^2+9}}$.

Sol

$\Rightarrow \int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{x^2\sqrt{x^2+(3)^2}}$

$\Rightarrow$ Put $x = 3\tan\theta$

$dx = 3\sec^2\theta d\theta$

When

$x = \sqrt{3}$

$\sqrt{3} = \sqrt{3}\sqrt{3}\tan\theta$

$\theta = \tan^{-1}\frac{1}{\sqrt{3}}$.

$\theta = \pi/6$.

$3\sqrt{3} = 3\tan\theta$

$\theta = \pi/3$

$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{3\sec^2\theta d\theta}{(3\tan^2\theta)\sqrt{9\tan^2\theta+9}}$

Ex 8.5

1 $\rightarrow$ 10

$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\cancel{3}\sec^{\cancel{2}}\theta \; d\theta}{9\tan^2\theta \; \cancel{3\sec\theta}}$

$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sec\theta \; d\theta}{9\tan^2\theta}$

$\Rightarrow \frac{1}{9}\int_{\pi/6}^{\pi/3} \frac{\sec\theta}{\tan^2\theta} d\theta$

$\Rightarrow \frac{1}{9}\int_{\pi/6}^{\pi/3} \sec\theta \; \frac{\cos^2\theta}{\sin^2\theta} \; d\theta$

-

$$\Rightarrow \frac{1}{9}\int_{\pi/6}^{\pi/3} \frac{1}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$\Rightarrow \frac{1}{9}\int_{\pi/6}^{\pi/3} \sin^{-2}\theta \cos\theta\, d\theta$$

$$\Rightarrow \frac{1}{9}\left[\frac{\sin^{-1}\theta}{-1}\right]_{\pi/6}^{\pi/3}$$

$$\Rightarrow -\frac{1}{9}\left[\frac{1}{\sin\theta}\right]_{\pi/9}^{\pi/3}$$

$$\Rightarrow -\frac{1}{9}\left[\frac{1}{\sin\pi/3} - \frac{1}{\sin\pi/6}\right]$$

$$\Rightarrow -\frac{1}{9}\left[\frac{2}{\sqrt{3}} - \frac{1}{1/2}\right]$$

$$\Rightarrow -\frac{1}{9}\left[\frac{2}{\sqrt{3}} - 2\right]$$

$$\Rightarrow -\frac{1}{9}\left[\frac{2 - 2\sqrt{3}}{\sqrt{3}}\right]$$

$$\Rightarrow -\frac{1}{9\sqrt{3}}\left[2 - 2\sqrt{3}\right]$$

Q 11 $\int \frac{x^3}{\sqrt{a^2-x^2}}\, d$

Put $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$$\Rightarrow \int \frac{(a\sin\theta)^3 \, a\cos\theta \, d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$$

$$\Rightarrow \int \frac{a^3 \sin^3\theta \; a\cos\theta \, d\theta}{a\cos\theta}$$

$$\Rightarrow a^3 \int \sin^3\theta \, d\theta$$

$$\Rightarrow a^3 \int \sin^2\theta \, \sin\theta \, d\theta$$

$$\Rightarrow a^3 \int (1-\cos^2\theta)\sin\theta \, d\theta$$

$$\Rightarrow a^3 \int \sin\theta \, d\theta - a^3\int \cos^2\theta \sin\theta \, d\theta$$

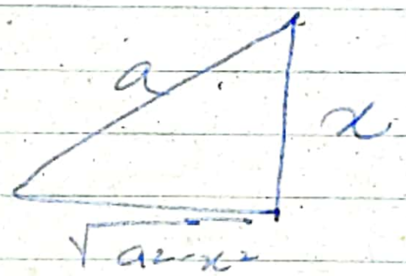
$$\Rightarrow a^3 \int \sin\theta \, d\theta + a^3\int \cos^2\theta \, (-\sin\theta) \, d\theta$$

$$\Rightarrow -a^3\cos\theta + \frac{a^3\cos^3\theta}{3} + c.$$

$$\Rightarrow \cancel{-a^3}$$

$x = a\sin\theta$

$\frac{x}{a} = \sin\theta$



$$\Rightarrow -\frac{a^3\sqrt{a^2-x^2}}{a} + \frac{a^3}{3}\left(\frac{\sqrt{a^2-x^2}}{a}\right)^3 + c$$

$$\Rightarrow -a^2\sqrt{a^2-x^2} + \frac{a^3}{3}\frac{\left(\sqrt{a^2-x^2}\right)^3}{a^3} + c$$

$$\Rightarrow -a^2\sqrt{a^2-x^2} + \frac{1}{3}(a^2-x^2)^{3/2} + c$$

Ans

Trigonometric substitution

① $\sqrt{a^2-x^2}$

Q 1 $\int_{-1}^{\sqrt{3}} \sqrt{4-x^2}\,dx$

Sol $\int_{-1}^{\sqrt{3}} \sqrt{(2)^2-x^2}\,dx$

Put $x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

When

$x=-1 \Rightarrow -1 = 2\sin\theta$

$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

$\theta = -\pi/6$

$x=\sqrt{3} \Rightarrow \sqrt{3} = 2\sin\theta$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \pi/3$

$\therefore \int_{-\pi/6}^{\pi/3} \sqrt{4-4\sin^2\theta}\;2\cos\theta\,d\theta$

$\Rightarrow 2\int_{-\pi/6}^{\pi/3} \sqrt{1-\sin^2\theta}\;2\cos\,d\theta$

$$\Rightarrow 4\int_{-\pi/6}^{\pi/3} \cos^2\theta \, d\theta$$

$$\Rightarrow \int_{-\pi/6}^{\pi/3} \left(\frac{1+\cos 2\theta}{2}\right) d\theta \qquad \because \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\Rightarrow \frac{4}{2}\int_{-\pi/6}^{\pi/3} (1+\cos 2\theta)\, d\theta$$

$$\Rightarrow 2\left[\theta + \frac{\sin\theta}{2}\right]_{\pi/6}^{\pi/3}$$

$$\Rightarrow 2\left\{\left[\pi/3 + \frac{\sin 2\pi/3}{2}\right] - \left[-\pi/6 + \frac{\sin 2(\pi/6)}{2}\right]\right\}$$

$$\Rightarrow 2\left\{\left[\pi/3 + \sqrt{3}/4\right] - \left[-\pi/6 + \frac{(-\sqrt{3}/2)}{2}\right]\right\}$$

$$\Rightarrow 2\left\{\left[\pi/3 + \frac{\sqrt{3}}{4} + \pi/6 + \sqrt{3}/4\right]\right\}$$

$$\Rightarrow 2\left\{\frac{4\pi + 3\sqrt{3} + 2\pi + 3\sqrt{3}}{12}\right]$$

$$\Rightarrow \frac{6\pi + 6\sqrt{3}}{6} = \frac{6(\pi+\sqrt{3})}{6}$$

$$\Rightarrow \pi + \sqrt{3}$$

5 (vi)

$$\int_{\pi/4}^{\pi/6} \cos^4 2x \; \sin^3 2x \, dx$$

Sol

$$\Rightarrow \int \cos^4 2x \cdot \sin^3 2x \, dx$$

$$\Rightarrow \int \cos^4 2x \; \sin^2 2x \; \sin 2x \, dx$$

$$\Rightarrow \int \cos^4 2x \, (1 - \cos^2 2x) \sin 2x \, dx$$

$$\Rightarrow \int \cos^4 2x \, (1 - \cos^2 2x) \; \sin 2x \, dx$$

same Q $\int \sec^4 2u\, du = ?$

Q $\int \tan^4 x\, dx$

Sol

$\Rightarrow \int \tan^2 x \tan^2 x\, dx$

$\Rightarrow \int \tan^2 x(\sec^2 x - 1)\, dx$

$\Rightarrow \int \tan^2 x \sec^2 x - \int \tan^2 x\, dx$

$\Rightarrow \int \tan^2 x \sec^2 x\, dx - \int (\sec^2 x - 1)\, dx$

$\Rightarrow \int \underset{f(x)}{\tan^2 x} \underset{f'(x)}{\sec^2 x}\, dx - \int \sec^2 x\, dx + \int dx$

$\Rightarrow \int \frac{\tan^{2+1} x}{3} - \tan x + x + c$

$\Rightarrow \frac{1}{3} \tan^3 x - \tan x + x + c$ Ans

Q $\int \sec^4 2u\, du \Rightarrow ?$

Sol $\int (\sec^2 2u)^2\, du$

$\Rightarrow \int \sec^2 2u \sec^2 2u\, du$

$\Rightarrow \int \sec^2 2u\, (1 + \tan^2 2u)\, du$

$\Rightarrow \int \sec^2 2u\, dx + \int \sec^2 2u \tan^2 2u\, du$

$\Rightarrow \frac{\tan 2u}{2} + \frac{\tan^3 2u}{3 \times 2} + c$

$\tan^4 2u\, du = ?$

dx

$x\, dx$

$x - 1)\, dx$

$dx + \int dx$

$x + c$

c Ans

du

du

$\tan^2 2u\, du$

c

$\frac{1}{2}\tan 2u + \frac{\tan^3 2u}{6} + c$

Q $\int \tan^5 x\, dx$

$\Rightarrow \int \tan^3 x \tan^2 x\, dx$

$\Rightarrow \int \tan^3 x (\sec^2 x - 1)\, dx$

$\Rightarrow \int \tan^3 x \sec^2 x\, dx - \int \tan^3 x\, dx$

$\Rightarrow \int \tan^3 x \sec^2 x\, dx - \int \tan^2 x \tan x\, dx$

$\Rightarrow \int \tan^3 x \sec^2 x\, dx - \int \tan x (\sec^2 x - 1)\, dx$

$\Rightarrow \int \tan^3 x \sec^2 x\, dx - \int \tan x \sec^2 x + \int \tan x\, dx$

$\Rightarrow \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \ln \sec x + c$

Q $\int \cos 2x\, dx$

$\Rightarrow \sin$

Q $\int_2^3 \sec$

Q18 $\int \tan^3 3x \cdot \sec^4 3x \, dx$

Sol

put $3x = u$.

$\Rightarrow 3dx = du$.

$\Rightarrow \frac{1}{3}\int \tan^3 u \sec^4 u \, du$.

$\Rightarrow \frac{1}{3}\int \tan^3 u \sec^2 u \sec^2 u \, du$

$\Rightarrow \frac{1}{3}\int \tan^3 u (1+\tan^2 u) \sec^2 u \, du$

$\Rightarrow \frac{1}{3}\int \tan^3 u \sec^2 u \, du + \frac{1}{3}\int \tan^5 u \sec^2 u \, du$

$\Rightarrow \frac{1}{3} \frac{\tan^4 u}{4} + \frac{1}{3} \frac{\tan^6 u}{6} + c$

$\Rightarrow \frac{1}{12} \tan^4 + \frac{1}{18} \tan^6 u + c$

Q $\int \tan^3 2x \cdot \sec^3 2x \, dx$

Sol

$\int \tan^3 2x \cdot \sec^3 2x \, dx$

Put $2x = u$

$2 dx = du$

$dx = \frac{du}{2}$

$\Rightarrow \frac{1}{2}\int \tan^3 u \sec^3 u \, du$

$\Rightarrow \frac{1}{2}\int \tan^2 u \cdot \tan u \cdot \sec^3 u \sec u \, du$

Q cot

$\Rightarrow \frac{1}{2}\int \tan u(\sec^2 u-1)\sec^3 u\,du$

$\Rightarrow \frac{1}{2}\int \tan u\sec^5 u\,du - \frac{1}{2}\int \tan u\sec^3 u\,du$

$\Rightarrow \frac{1}{2}\int \sec^4 u\tan u\sec u\,du - \frac{1}{2}\int \sec u^2 \sec u\tan u\,du$

$\Rightarrow \frac{1}{2}\frac{\sec^5 u}{5} - \frac{1}{2}\frac{\sec^3 u}{3} + c$

$\Rightarrow \frac{1}{10}\sec^5 2x - \frac{1}{6}\sec^3 2x + c$

Q(xiii) $\int \cot^4 3z\,dz$

Sol

Put $3z = u$

$3dz = du$

$\Rightarrow \frac{1}{3}\int \cot^4 u\,du$

$\Rightarrow \frac{1}{3}\int \cot^2 u\cot^2 u\,du$

$\Rightarrow \frac{1}{3}\int \cot^2 u(\operatorname{cosec}^2 u-1)\,du$

$\Rightarrow \frac{1}{3}\int \cot^2 u\operatorname{cosec}^2 u\,du - \frac{1}{3}\int \cot^2 u\,du$

$\Rightarrow \frac{1}{3}\int \cot^2 u\operatorname{cosec}^2 u\,du - \frac{1}{3}\int(\operatorname{cosec}^2 u-1)\,du$

$\Rightarrow \frac{1}{3}\int \cot^2 u\operatorname{cosec}^2 u\,du - \frac{1}{3}\operatorname{cosec}^2 u + \frac{1}{3}\int du$

$\Rightarrow \frac{1}{3}\int \cot^2 u(-\operatorname{cosec}^2 u)\,du - \frac{1}{3}\int \operatorname{cosec}^2 u + \frac{1}{3}\int du$

$\Rightarrow -\frac{1}{3}\ \frac{\cot^3 u}{3} - \frac{(-\cot^2 u)}{3} + \frac{1}{3}u$

$\sec^3 u\, du$

$= -\frac{1}{9}\cot^3 u + \frac{\cot^2 u}{3} + \frac{1}{3}u$

$\tan u \sec^3 u\, du$

$\Rightarrow -\frac{1}{9}\cot^3 3z + \frac{1}{3}\cot^2 3z + \frac{1}{3}(3z)$

$\frac{1}{2}\int \sec u^2 \sec u \tan u\, du$

by 21, 25

${}^3 u + c$

8.4

Q8 $\int_{\pi/3}^{\pi/6} \sin^2 x \cos x\, dx$

$^2 2x + c$

Sol

$\Rightarrow \left[\frac{\sin^3 x}{3}\right]_{\pi/3}^{\pi/6}$

$\Rightarrow \frac{1}{3}\left\{(\sin \pi/6)^3 - (\sin \pi/3)^3\right\}$

$\Rightarrow \frac{1}{3}\left\{\left(\frac{1}{2}\right)^3 - \left(\frac{\sqrt{3}}{2}\right)^3\right\}$

du

$\Rightarrow \frac{1}{3}\left\{\frac{1}{8} - \frac{3\sqrt{3}}{8}\right\}$

$u - 1)\, du$

$\Rightarrow \frac{1}{3}\left\{\frac{1 - 3\sqrt{3}}{8}\right\}$

$du - \frac{1}{3}\int \cot^2 u\, du$

$= \frac{1}{24}\left\{1 - 3\sqrt{3}\right\}$ Ans

$\frac{1}{3}\int (\operatorname{cosec}^2 u - 1)\, du$

Q9 $\int_0^{\pi/2} \cos^4 x\, dx$

$\operatorname{cosec}^2 u + \frac{1}{3}\int du$

$\operatorname{cosec}^2 u + \frac{1}{3}\int du$

$\Rightarrow \int_0^{\pi/2} \cos^2 (1 - \sin^2 x)\, dx$

$$\Rightarrow \int_0^{\pi/2} \cos^2 x \, dx - \int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$$

Q10 $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\cos^3 x}{\sqrt{\sin x}} \, dx$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos^3 x \, \sin x^{-1/2} \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos^2 x \cos x \, \sin^{-1/2} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos x (1 - \sin^2 x) \sin^{-1/2} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos x \, \sin^{-1/2} x \, dx - \int_{\pi/6}^{\pi} \sin^{3/2} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos x \, \sin^{-1/2} x \, dx - \int_{\pi/6}^{\pi/2} \sin^{3/2} x \, dx$$

$$\Rightarrow \left[\frac{\sin^{1/2} x}{1/2} \right]_{\pi/6}^{\pi/2} - \left[-\cos^{3/2} x \right]_{\pi/6}^{\pi/2}$$

$$\Longrightarrow \frac{1}{2}\left\{(\sin \pi/2)^{1/2} - (\sin \pi/6)^{1/2}\right] + \left[(\cos \pi/2)^{3/2} - (\cos \pi/6)^{3/2}\right]$$

$$\Longrightarrow \frac{1}{2}\left[0 - \left(\frac{\sqrt{3}}{2}\right)^{1/2}\right] + \left[\cos^{3/2} - (1/\sqrt{2})^{3/2}\right]$$

$$\Longrightarrow \frac{1}{2}\left\{-\frac{3}{\sqrt{2}}\right\} + \left\{1 - \frac{1}{4}\right]$$

$$\Longrightarrow -\frac{3}{2\sqrt{2}} + \frac{3}{4}$$

$$\Longrightarrow \frac{-3(4) + 3(2\sqrt{2})}{2\sqrt{2}(4)} \Longrightarrow \frac{-12 + 6\sqrt{2}}{8\sqrt{2}}$$

$\Longrightarrow$

Q 10 $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\cos^3 x}{\sqrt{\sin x}}\, dx = ?$

Sol

$$\int_{\pi/6}^{\pi/2} \cos^2 x \cos x (\sin x)^{-1/2}\, dx$$

$$\Longrightarrow \int_{\pi/6}^{\pi/2} (1 - \sin^2 x) \sin^{-1/2} x \cdot \cos x\, dx$$

$$\Longrightarrow \int_{\pi/6}^{\pi/2} (\sin^{-1/2} x - \sin^{3/2} x) \cos x\, dx$$

$$\Longrightarrow \int_{\pi/6}^{\pi/2} \sin x^{-1/2} \cos x\, dx - \int_{\pi/6}^{\pi/2} \sin^{3/2} x \cos x\, dx$$

$$\longrightarrow \left[\frac{\sin^{1/2} x}{1/2}\right]_{\pi/6}^{\pi/2} - \left[\frac{\sin^{5/2} x}{5/2}\right]_{\pi/6}^{\pi/2}$$

$$\Longrightarrow 2\left[\sin^{1/2} x\right]_{\pi/6}^{\pi/2} - \frac{2}{5}\left[\sin^{5/2} x\right]_{\pi/6}^{\pi/2}$$

$$\Rightarrow 2\left[(\sin \pi/2)^{1/2} - (\sin \pi/6)^{1/2}\right] - \frac{2}{5}\left[(\sin \pi/2)^{5/2} - (\sin \pi/6)^{5/2}\right]$$

$$\Rightarrow 2\left[(1)^{1/2} - \left(\tfrac{1}{2}\right)^{1/2}\right] - \frac{2}{5}\left[(1)^{5/2} - \left(\tfrac{1}{2}\right)^{5/2}\right]$$

$$\Rightarrow 2 - \frac{2}{\sqrt{2}} - \frac{2}{5} + \frac{2}{5}\cdot\frac{1}{4\sqrt{2}}$$

$$\Rightarrow 2 - \sqrt{2} - \frac{2}{5} + \frac{1}{5\cdot 2\sqrt{2}}$$

$$\Rightarrow 2 - \sqrt{2} - \frac{2}{5} - \frac{1}{10\sqrt{2}}$$

$$\Rightarrow \boxed{\frac{8}{5} - \sqrt{2} - \frac{1}{10\sqrt{2}}}$$

Q11 Q $\int_{0}^{\pi/} \tan^3 x \sec x \, dx$

$$\Rightarrow \int_{0}^{\pi/} (\sec^2 x - 1)\tan x \sec x \, dx$$

$$\Rightarrow \int_{0}^{\pi/} (\sec^3 x - \sec x)\tan x \, dx$$

$$\Rightarrow \int_{0}^{\pi/} \sec^3 x \tan x \, dx - \int_{0}^{\pi/} \sec x \tan x \, dx$$

$$\Rightarrow \int_{0}^{\pi} \sec^2 x \sec x \tan x \, dx - \int_{0}^{\pi} \sec x \tan x \, dx$$

$$\Rightarrow \left[\frac{\sec^3 x}{3}\right]_0^{\pi} - \left[\sec x\right]_0^{\pi}$$

$$\Rightarrow \frac{1}{3}\left[\sec^3 x\right]_0^{\pi} - \left[\sec x\right]_0^{\pi}$$

$\cos 180° = -1$
$\therefore \sec\pi = -1$

$$\Rightarrow \frac{1}{3}\left[(\sec\pi)^3 - (\sec 0)^2\right] - \left[(\sec\pi)^3 - (\sec 0)\right]$$

$$\Rightarrow \frac{1}{3}\left[(-1)^3 - (1)^2\right] - \left[-1 - 1\right]$$

$$\Rightarrow + \frac{1}{3}(-2) + 2$$

$$\Rightarrow -\frac{2}{3} + 2$$

$$\Rightarrow \boxed{\frac{4}{3}} \text{ Ans}$$

Q $\int_0^{\pi/4} \sin^2 2x \cos^2 2x \, dx = ?$

$$\Rightarrow \int_0^{\pi/4} \sin^2 2x \cos^2 2x \, dx$$

Let

$$2x = u$$

$$\Rightarrow 2dx = du$$

$$\Rightarrow \frac{1}{2} \int_0^{\pi/4} \sin^2 u \cos^2 u \, du$$

$$\Rightarrow \frac{1}{2} \int_0^{\pi/4} \left(\frac{1-\cos 2u}{2}\right)\left(\frac{1+\cos 2u}{2}\right) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} (1 - \cos^2 2u) \, du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} du - \frac{1}{8} \int_0^{\pi/4} \cos^2 2u \, du$$

$$\cancel{\Rightarrow \frac{1}{8} [u]_0^{\pi/4} - \frac{1}{8} [}$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} du - \frac{1}{8} \int_0^{\pi/4} \left(\frac{1+\cos 4u}{2}\right) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} du - \frac{1}{16} \int_0^{\pi/4} du -$$

$$\Rightarrow \frac{1}{8}\int_0^{\pi/4}\left(1-\frac{1-\cos 4u}{2}\right)du$$

$$\Rightarrow \frac{1}{8}\int_0^{\pi/4}\left(\frac{2-1-\cos 4u}{2}\right)du$$

$$\Rightarrow \frac{1}{8}\int_0^{\pi/4}\left(\frac{1-\cos 4u}{2}\right)du$$

$$\Rightarrow \frac{1}{16}\int_0^{\pi/4}du-\frac{1}{16}\int_0^{\pi/4}\cos 4u\,du$$

$$\Rightarrow \frac{1}{16}\left[u\right]_0^{\pi/4}-\frac{1}{16}\left[\frac{\sin 4u}{4}\right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{16}\left[u\right]_0^{\pi/4}-\frac{1}{64}\left[\sin 4u\right]_0^{\pi/4}$$

$$\Rightarrow \cancel{\frac{1}{16}\left[\frac{\pi}{4}\right.}$$

$$\Rightarrow \frac{1}{16}\left[2x\right]_0^{\pi/4}-\frac{1}{64}\left[\sin 8x\right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{16}\left[2\pi\right.$$

8.7.

Integration by Parts:-

$$\int (I)(II)\,dx = (I)\cdot\left(\text{Integral of } II\right) - \int \left(\text{Derivative of } I\right)\left(\text{Integral of } II\right)dx$$

$$\int u\cdot v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

Q: $\int x\sin x\,dx$

Sol: Using by parts.

$$\int u\cdot v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$\Rightarrow x(-\cos x) - \int\left\{\frac{d}{dx}(x)\int \sin x\,dx\right\}dx$$

$$\Rightarrow -x\cos x - \int\{1(-\cos x)\}dx$$

$$\Rightarrow -x\cos x + \int \cos x\,dx$$

$$\Rightarrow -x\cos x + \sin x + c$$ Ans

Q $\int x\sin x\,dx$

Using parts:-

$$\int u\cdot v\,dx = u\int v\,dx \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

Put $u = x$ and $v = \sin x$

$\frac{du}{dx} = 1$ and $\int v\,dx = \int \sin x\,dx = -\cos x$

$$\int x\sin x\,dx = x\cdot(-\cos x) \int 1\cdot(-\cos x)\,dx$$

Liate.

$L = \ln$

I = Inverse. Trig

A = Algebric.

T = Trignometric

$e = e^x$

Note:- Inverse Trignometric func. and logrithm func. can be taking as second function in by parts formula.

Q $\int \ln x \, dx$

$\Rightarrow \int 1 \cdot \ln x \, dx$

$\Rightarrow \int \ln x \cdot 1 \, dx$

Using $\int u \cdot v \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \int v \, dx \right\} dx$

Taking

$u = \ln x$; $v = 1$

$\frac{du}{dx} = \frac{1}{x}$; $\int v \, dx = \int dx$

$\therefore \int v \, dx = x$

$\therefore \int \ln x \cdot 1 \, dx = \ln x \cdot x - \int \left(\frac{1}{x} \cdot x \right) dx$

$= \ln x \cdot x - \int dx$

$\int \ln x \, 1 \, dx = x \ln x - x + C$

c. Trig
er.
tri

tric func
g. as.
parts
e

$\{\int v\,dx\}dx$

1
dx
x

$x)\,dx$

8.7

Q $\int x \ln x \, dx$

$\rightarrow \int \ln x \cdot x \, dx$

$\Rightarrow \int \ln x \cdot x \, dx$

Let

$u = \ln x \qquad v = x$

$\frac{du}{dx} = \frac{1}{x} \quad ; \int v\,dx = \int x\,dx = \frac{x^2}{2}$

Using by parts.

$\int \ln x \cdot x\,dx = \ln x\left(\frac{x^2}{2}\right) - \int\left(\frac{1}{x}\frac{x^2}{2}\right)dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$

$\Rightarrow \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + c$

$\Rightarrow \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$

Q $\int x^2 \tan^{-1}x\,dx$

Using by parts:- $\int u \cdot v\,dx = u\int v\,dx - \int\{\frac{du}{dx}\int v\,dx\}dx$

$u = \tan^{-1}x \qquad ; \quad v = x^2$

$\frac{du}{dx} = \frac{1}{1+x^2} \qquad \int v\,dx = \int x^2\,dx = \frac{1}{3}x^3$

$\int x^2 \tan^{-1}x\,dx = \tan^{-1}x\left(\frac{x^3}{3}\right) - \int\frac{1}{1+x^2}\left(\frac{x^3}{3}\right)d$

$\Rightarrow \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int\frac{x^3}{1+x^2}\,dx$

Q4, 5, 6, 7, 8,

$$1+x^2 \overline{) \, x^3 + \,} \quad x$$
$$\underline{-x^3 + 1}$$
$$-1$$

$$\Rightarrow \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$$

$$\Rightarrow \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int x\,dx + \frac{1}{3}\int \frac{x}{1+x^2}\,dx$$

$$\Rightarrow \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int x\,dx + \frac{1}{3\times 2}\int \frac{2x}{1+x^2}\,dx$$

$$\Rightarrow \frac{x^3}{3}\tan^{-1}x - \frac{1}{3\times 2}x^2 + \frac{1}{6}\ln(1+x^2) + c$$

Q 1 (ii) $\int x\sin^2 x\cos x\,dx$

Sol:

Let $u = \cancel{x}$; $v = \sin^2 x\cos x$

$\frac{du}{dx} = \cancel{dx}$ $\quad \int v\,dx = \int \sin^2 x\cos x\,dx$

$$\int v\,dx = \frac{\sin^3 x}{3}$$

Using by parts:

$$\int u.v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$\therefore \int x\sin^2 x\cos x\,dx = x\,\frac{\sin^3 x}{3} - \int\left(1\cdot\frac{\sin^3 x}{3}\right).dx$$

$$= \frac{x}{3}\sin^3 x - \frac{1}{3}\int \sin^3 x\,dx$$

(2) $\int 1 \cdot dx \ldots$

Q $\int (\ln x)^2 dx$

$\Rightarrow \int (\ln x)^2 \cdot 1\, dx$

Let

$u = (\ln x)^2$; $v = 1$

$\frac{du}{dx} = \frac{2\ln x}{x}$ $\int v\,dx = \int 1\,dx = x$

Using

$\int u.v\,dx = u\int v\,dx - \int \left\{ \frac{du}{dx} \int v\,dx \right\} dx$

$\int (\ln x)^2 \cdot 1\,dx = (\ln x)^2 \cdot x - \int \left(\frac{2\ln x}{x} \cdot x \right) dx$

$= (\ln x)^2 x - 2\int \ln x\,dx$

$= (\ln x)^2 \cdot x - 2\left[\ln x \cdot x - \int \left(\frac{1}{x} \right) x\, dx \right]$

$= (\ln x)^2 x - 2[x \ln x - x] + c$

$= (\ln x)^2 x - 2x\ln x + 2x + c$

Theorem = Every integral can be solved by integration by Parts.

Q^2 (i) $\int \sqrt{4-x^2}\,dx$

$\Rightarrow \int 1\,\sqrt{4-x^2}\,dx$

$\Rightarrow$ Let

$u = 1 \qquad v = \sqrt{4-x^2}$

$\frac{du}{dx} = 0 \qquad \int v\,dx = \int \sqrt{4-x^2}\,dx.$

Using

$\int u.\,v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}.dx$

$= 1.\int \sqrt{4-x^2}\,dx \int$

Q $\int x^2 e^x\,dx = ?$

Sol Let

$u = x^2 \quad ; \quad v = e^x$

$\frac{du}{dx} = 2x. \quad \int v\,dx = \int e^x\,dx = e^x.$

Q $\int x e^x dx$

Let $u = x$, $v = e^x$

$\frac{du}{dx} = 1$ $\int v dx = \int e^x dx = e^x$

$\int x e^x dx = x e^x - \int 1 . e^x dx$

$= x e^x - \int e^x dx$

$= x e^x - e^x + c$

$= e^x (x - 1) + c$

Q $\int \sqrt{\sin \sqrt{2x}}\, dx$

Put

$\sqrt{2x} = t$

$2x = t^2$

$\cancel{2}t \frac{dt}{dx} = \cancel{2}$

$t . dt = dx$

$\Rightarrow \int \sin t . t dt$

$\Rightarrow \int t \sin t . dt$

Let

$t = u$ $v = \sin t$

$\frac{dt}{du} = 1$ $\int v dx = \int \sin t dt = -\cos t$

Using

$\int u . v dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$

$$\int t \sin t \, dt = t(-\cos t) - \int 1.(-\cos t) \, dt$$
$$= -t\cos t + \int \cos t \, dt$$
$$= -t\cos t + \sin t + c.$$

$$\int \sin\sqrt{2x} \, dx = -\sqrt{2}x \cos\sqrt{2x} + \sin\sqrt{2x} + c$$

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Ex 4// Pg 238.

$$\int e^{ax} \sin bx \, dx = ?$$

Sol:

Let

$e^{ax} = u$ $\qquad$ $v = \sin bx$

$ae^{ax} = \frac{du}{dx}$ $\qquad$ $\int v \, dx = -\frac{\cos bx}{b}.$

Using

$$\int u.v \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \int v \, dx \right\} dx$$

$$\therefore \int e^{ax} \sin bx \, dx = e^{ax}\left(-\frac{\cos bx}{b}\right) - \int \left(ae^{ax}\left(-\frac{\cos bx}{b}\right)\right) dx$$

$$= -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \int e^{ax} \cos bx \, dx$$

consider $I = \int e^{ax} \sin bx.$

$$I = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \int e^{ax} \cos bx \, dx$$

$$I = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b}\left[e^{ax} \frac{\sin bx}{b} - \int ae^{ax} \frac{\sin bx}{b} \, dx\right]$$

$$I = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b^2} e^{ax} \sin bx - \frac{a^2}{b^2} \int e^{ax} \sin bx \, dx$$

$$= -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b^2} e^{ax} \sin bx - \frac{a^2}{b^2} I$$

(xii) (xiii).

3 Q 12, 13

$$I + I\frac{a^2}{b^2} = -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b^2}e^{ax}\sin bx$$

$$I\left(\frac{a^2+b^2}{b^2}\right) = \frac{-be^{ax}\cos bx + ae^{ax}\sin bx}{b^2}$$

$$I = \frac{e^{ax}}{a^2+b^2}\left[-b\cos bx + a\sin x\right]$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right] \text{ Ans}$$

Q 4 Calculate ?

(i) $\int e^x(\sin x + \cos x)dx = ?$

Using

$$\int e^{ax}[af(x) + f'(x)]dx = e^{ax}f(x) + c$$

$$\therefore \int e^x(\sin x + \cos x)dx = e^x\sin x + c \text{ Ans}$$

Q $\int e^x\left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right)dx = ?$

Using

Q $\int e^x\left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2}\right]dx$

$$= e^x\frac{1}{(1-x)} + c$$

$y = \frac{1}{1-x} = (1-x)^{-1}$

$= (-1)(x-1)^{-2}\frac{d}{dx}(1-x)$

$y = \frac{(-1)}{(x-1)^2}(-1) = \frac{1}{(x-1)^2}$

4 (iii) $\int \frac{x e^x}{(1+x)^2} dx \to ?$

$\Rightarrow \int \frac{x e^x}{(1+x)^2} dx$

$\Rightarrow \int \frac{e^x (1+x-1)}{(1+x)^2} dx$

$\Rightarrow \int e^x \left[\frac{1+x}{(1+x)^2} - \frac{1}{(1+x)^2} \right] dx$

$\Rightarrow \int e^x \left[\frac{1+x}{(1+x^3)^2} + \frac{(-1)}{(1+x)^2} \right] dx$

Using form

$\Rightarrow e^x \frac{1}{(1+x)} + c$ $\quad \int e^{ax} [a f(x) f'(x)] dx = e^{ax} f(x) + c$

$\Rightarrow \frac{e^x}{1+x} + c$

Q 4 (vi) $\int \cos \left(b \ln \frac{x}{a} \right) dx$;

Sol

$\frac{x}{a} = e^u$

$x = ~~du~~ \; a e^u$

$dx = a e^u du.$

$\cos b \ln \frac{x}{a} = ~~\cos~~$

$= \cos b \ln (e^u)$

$= \cos bu$

$\int \cos \left(b \ln \frac{x}{a} \right) = \cos bu \, a e^u du$

$= a \int \cos bu \, e^u du$

$\cos^5 x$

$\cos^4 x \left(\sin^2 x - 1\right)$

a, e^{u}

$\int \cos x (1 - \sin^2 x)^2 \, dx$

$\int \cos x \, dx - \int \sin^2 x \cos x \, dx$

$\sin x - \frac{\sin^3 x}{3} + c$

$\int \cos x (1 + \sin^4 x - 2\sin^2 x) \, dx$

$\int \cos x \, dx + \int \sin^4 x \cos x \, dx - 2\int \sin^2 x \cos x \, dx$

$\rightarrow \sin x + \frac{\sin^5 x}{5} - \frac{2}{3} \sin^3 x + c$

$$\Rightarrow \int_1^2 (x^2+x+1)^{1/3} (2x+1)\,dx$$

$$\Rightarrow \left[\frac{(x^2+x+1)^{4/3}}{4/3}\right]_1^2$$

$$\Rightarrow \frac{3}{4}\left[(x^2+x+1)^{4/3}\right]_1^2$$

$$\Rightarrow \frac{3}{4}\left\{((2)^2+2+1)^{4/3} - (1^2+1+1)^{4/3}\right\}$$

$$\Rightarrow \frac{3}{4}\left[7^{4/3} - 3^{4/3}\right] \qquad \sqrt[3]{7\times7\times7\times7}$$

$$\Rightarrow \frac{3}{4}\left\{7\sqrt[3]{7} - 3\sqrt[3]{3}\right] \qquad 7\sqrt[3]{7}$$

$\Rightarrow$

Q 7 $\int_{-2}^{1} (x^2+1)^{1/2}\, x\,dx$

$$\Rightarrow \frac{1}{2}\int_{-2}^{1} (x^2+1)^{1/2}\, 2x\,dx$$

$$\Rightarrow \int_{-2}^{1} \frac{1}{2}\left[\frac{(x^2+1)^{3/2}}{3/2}\right]_{-2}^{1}$$

$$\Rightarrow \frac{2}{3}\times\frac{1}{2}\left[(x^2+1)^{3/2}\right]_{-2}^{1}$$

$$\Rightarrow \frac{2}{3}\times\frac{1}{2}\left\{(1^2+1)^{3/2} - \{(-2)^2+1\}^{3/2}\right\}$$

$$\Rightarrow \frac{1}{3}\left\{2\sqrt{2} - 5\sqrt{5}\right\}.$$

Example pg 215

Q $\int \frac{du}{\sqrt{u+a}+\sqrt{u+b}} = ?$

$\Rightarrow \int \frac{du}{\sqrt{u+a}+\sqrt{u+b}} \times \frac{\sqrt{u+a}-\sqrt{u+b}}{\sqrt{u+a}-\sqrt{u+b}}$

$\Rightarrow \int \frac{du(\sqrt{u+a}-\sqrt{u+b})}{(u+a)-(u+b)}$

$\Rightarrow \int \frac{(\sqrt{u+a}-\sqrt{u+b})\,du}{u+a-u-b}$

$\Rightarrow \frac{1}{a-b}\int (\sqrt{u+a}-\sqrt{u+b})\,du$

$\Rightarrow \frac{1}{a-b}\left[\int \sqrt{a+u}\,du - \int \sqrt{u+b}\,du\right]$

$\Rightarrow \frac{1}{a-b}\left[\frac{(a+u)^{3/2}}{3/2} - \frac{(u+b)^{3/2}}{3/2}\right]+c$

$\Rightarrow \frac{1}{a-b}\left[\frac{2(a+u)^{3/2}}{3} - \frac{2}{3}(u+b)^{3/2}\right]+c$

$\Rightarrow \frac{2}{3(a-b)}\left\{(a+u)^{3/2}-(u+b)^{3/2}\right\}+c$ Ans

Exmp pg-217

Q $\int_0^1 \frac{dx}{(2x+3)^{2/3}} = ?$

Sol

$\int_0^1 \frac{(dx)}{(2x+3)^{2/3}}$

$$\Rightarrow \int_0^1 (2x+3)^{-2/3}\, dx$$

$$\Rightarrow \frac{1}{2}\int_0^1 (2x+3)^{-2/3} \cdot 2\, dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{(2x+3)^{1/3}}{1/3}\right]_0^1$$

$$\Rightarrow \frac{3}{2}\left[(2x+3)^{1/3}\right]_0^1$$

$$\Rightarrow \frac{3}{2}\left\{(2(1)+3)^{1/3} - (2\cdot 0+3)^{1/3}\right\}$$

$$\Rightarrow \frac{3}{2}\left\{\sqrt[3]{5} - \sqrt[3]{3}\right\}$$ Ans

Q Evalute :- 2004

Q $\int_0^2 (x^2+3x+5)^{-2/3}\left(x+\frac{3}{2}\right) dx$

Sol

$$\Rightarrow \frac{1}{2}\int_0^2 (x^2+3x+5)^{-2/3}(2x+3)\, dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{(x^2+3x+5)^{1/3}}{1/3}\right]_0^2$$

$$\Rightarrow \frac{3}{2}\left\{(2^2+3\cdot 2+5)^{1/3} - (0^2+0+5)^{1/3}\right\}$$

$$\Rightarrow \frac{3}{2}\left\{\sqrt[3]{15} - \sqrt[3]{5}\right\}$$ Ans

$\int \sec x \, dx = \ln(\sec x + \tan x) + c$

own

Q $\displaystyle\int \frac{\sec\sqrt{x}}{\sqrt{x}}\,dx$

Sol

Let $\sqrt{x} = u$

$\cancel{\frac{1}{2}\sqrt{x}\,dx = du}$

$\frac{1}{2\sqrt{x}}dx = du$

$\frac{1}{\sqrt{x}}dx = 2du$

$\Rightarrow \int \sec u \, 2du$

$\Rightarrow 2\int \sec u \, du$

$\Rightarrow 2\{\ln(\sec x + \tan x)\} + c$

Q1-(xiii) $\int (\sec 4x - 1)^2 \, dx$

Sol

$\Rightarrow \int (\sec^2 4x - 2\sec 4x + 1)dx$

$\Rightarrow \int \sec^2 4x \, dx - 2\int \sec 4x + \int dx$

$\Rightarrow \frac{\tan 4x}{4} - 2\,\frac{\ln(\sec 4x + \tan 4x)}{4} + x$

$\int \sec^2 \boxed{3x} \, dx \Rightarrow \frac{\tan \square}{\square'} + c$

$\int \cos y \, dy = \frac{\sin \square}{\square'} + c$

$$\Rightarrow \frac{\tan 4x}{4} - \frac{1}{2}\left(\ln(\sec 4x + \tan 4x)\right) + x + c$$

$$\Rightarrow \frac{\tan 4x}{4} - \frac{1}{2}\ln(\sec 4x + \tan 4x)^{\frac{1}{2}} + x + c$$

$$\Rightarrow \frac{\tan 4x}{4} - \ln\left[(\sec 4x + \tan 4x)^{\frac{1}{2}}\right] + x + c$$

Ans

8 $\int \frac{dx}{\operatorname{cosec} 2x - \cot 2x}$

Sol:

$$\Rightarrow \int \frac{1 \quad dx}{\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}}$$

$$\Rightarrow \int \frac{dx}{\left(\frac{1 - \cos 2x}{\sin 2x}\right)}$$

$$\Rightarrow \int \frac{\sin 2x}{1 - \cos 2x} \, dx$$

Let.

$$1 - \cos 2x = u$$

$$0 + 2\sin 2x \, dx = du$$

$$\sin 2x \, dx = \frac{du}{2}$$

$$\Rightarrow \int \frac{1}{u} \cdot \frac{du}{2}$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{u} \cdot du$$

$$\Rightarrow \frac{1}{2}\ln u + c$$

$\Rightarrow \frac{1}{2} \ln(1-\cos 2x) + c$

$\Rightarrow \ln\sqrt{1-\cos 2x} + c$

Ans

H.W

Q $\int \frac{dx}{\operatorname{cosec} 5x - \cot 5x}$ $\Rightarrow \frac{1}{2} \ln(\cancel{2}\sin^2 x) + c$

$\Rightarrow \ln \sin^2 x + c$

Q. $\int \frac{dx}{\operatorname{cosec} 9x - \cot 9x}$

$\sin^2 \boxed{x} = \frac{1-\cos 2\boxed{\ }}{2} + c$ → Alone

$\sin^2 \boxed{x} . \sin x = 1 - \cos^2 \boxed{\ } + c$ → N. Alone

$\cos^2 \boxed{x} = \frac{1+\cos 2\boxed{\ }}{2} + c$ → Alone

$\cos^2 \boxed{x} . \cos x = 1 - \sin^2 \boxed{\ } + c$ → not. Alone

Q 5(i) $\int \sin^2 x \, dx$

$\longrightarrow \int \left(\frac{1-\cos 2x}{2}\right) dx$

$\longrightarrow \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x \, dx$

Using $\int \cos \square \, dx = \frac{\sin \square}{\square'} + c$

$\Rightarrow \frac{1}{2}x - \frac{\sin 2x}{2}\left(\frac{1}{2}\right) + c$

$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4} + c$

Ans

Q $\int \cos^2 x \, dx$

Sol

$\Rightarrow \int \left(\frac{1+\cos 2x}{2}\right) dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x \, dx$

$\Rightarrow \frac{x}{2} + \frac{1}{4}\sin 2x + c$

Q $\int \cos^2(3x+2) \, dx$

Sol $\Rightarrow$ Put

$3x+2 = u$

$3\,dx = du$

$dx = \frac{du}{3}$

$\Rightarrow \int \cos^2 u \, \frac{du}{3}$

$\Rightarrow \frac{1}{3}\int \cos^2 u \, du$

$\Rightarrow \frac{1}{3}\int \left(\frac{1+\cos 2u}{2}\right) du$

$\Rightarrow \frac{1}{6}\int du + \frac{1}{6}\int \cos 2u \, du$

$\Rightarrow \frac{u}{6} + \frac{1}{6}\left(\frac{\sin 2u}{2}\right) + c$

$$\Rightarrow \frac{\cancel{2x}\,(3x+2)}{6} + \frac{1}{12} \sin 2(3x+2) + c$$

$$= \frac{(3x+2)}{6} + \frac{1}{12} \sin 2(3x+2) + c$$

Ans

$$\int \sin^2 x \cos^2 x \, dx$$

$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

$$\frac{1}{4}\int (1-\cos 2x)(1+\cos 2x) \, dx$$

$$\frac{1}{4}\int (1-\cos^2 2x) \, dx$$

$$\frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x \, dx \qquad \frac{1+\cos 2\square}{2} = \cos^2 x$$

$$\frac{x}{4} - \frac{1}{4}\int \frac{1+\cos 4x}{2} \, dx$$

$$= \frac{x}{4} - \frac{1}{8}\int 1\,dx + \cos 4x \, dx$$

$$= \frac{x}{4} - \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4x \, dx.$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{8}\,\frac{\sin 4x}{4} + c \qquad \int \cos \square \, dx = \frac{\sin \square}{\square'} + c$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{32} \sin 2x + c$$ Ans.

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$\Rightarrow \int (1-\cos^2 x) \sin x \, dx$

$\Rightarrow \int \sin x \, dx - \int \sin x \cos^2 x \, dx$

$\Rightarrow -\cos x + \int (-\sin x) \cos^2 x \, dx$

$\Rightarrow -\cos x + \left[\frac{\cos^3 x}{3} \right] + c$

$\Rightarrow \frac{1}{3} \cos 3x - \cos x + c$

Ans

Q $\int \cos^3 x \, dx$

Sol

$\Rightarrow \int \cos^2 x \cos x \, dx$

$\Rightarrow \int (1-\sin^2 x) \cos x \, dx$

$\Rightarrow \int \cos x \, dx - \int \cos x \sin^2 x \, dx$

$\Rightarrow \sin x - \frac{\sin 3}{3} + c$

Q $\int \frac{\cos^3 y}{3} \, dy$

Sol $\int \frac{\cos^3 y}{3} \, dy$

$\Rightarrow \frac{1}{3} \int \cos^3 \left(\frac{y}{3}\right) dy$

$\Rightarrow \frac{1}{3}$

$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

Q5 (V) $\int \sin^2 x \cos^3 x \, dx$

Sol

$\longrightarrow \int \sin^2 x \, . \, \cos^2 x \, . \, \cos x \cdot dx$ (→ Ah, → N.Ah)

$\Rightarrow \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$

$\Rightarrow \int \sin^2 x \cos x \, dx - \int \sin^4 x \cos x \, dx$

$\Rightarrow \int$